Fluctuation Phenomena in Low Dimensional Conductors

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Chapter 1

Introduction

Mesoscopic physics is a subfield of condensed matter physics devoted to electrical phenomena of small conductors in which the quantum nature of the electrons plays an important role. The term 'meso' indicates that mesoscopic physics is at the borderline between the microscopic and macroscopic world. On one hand, the systems of interest are small enough, that electrons maintain their quantum phase coherence over a distance larger than the sample size leading to interference effects which cannot be described classically. The wavelength of the electrons can be of the same order as the confining potential so that quantized states are formed. On the other hand, mesoscopic systems contain like macroscopic devices a large number of atoms so that statistical descriptions like distribution functions are meaningful. The most important length scales defining the mesoscopic regime are the coherence length l_{ϕ} and the Fermi wavelength λ_F of the electrons. Although some mesoscopic effects are observable on a macroscopic scale - as for example the quantum Hall effect - most studies are carried out on devices of sub-micrometer dimensions. Thus, the experimental research in mesoscopic physics, which started around twenty years ago, has always closely been related with the development of sophisticated lithographic and crystal growth techniques. The fundamental research also benefitted much from the massive industrial research and development efforts towards miniturization of integrated electronic circuits based on semiconducting materials. Since nowadays these conventional techniques seem to reach their limits in further miniturization other materials such as organic molecules, nanotubes or DNA might be used in the future. That is why mesoscopic physics has also become a part of an interdisciplinary field including physics, chemistry and biology.

Initially, the research in mesoscopic physics was mainly focused on diffusive metals in which the electronic motion is a random walk between impurities. Interference in multiple scattering processes gives rise to corrections to

1 Introduction



Figure 1.1: Time dependent fluctuations $\Delta I(t) = I(t) - \langle I \rangle$ of the electrical current I(t) around its mean value $\langle I \rangle$.

the classical conductance as manifest in, for example, universal conductance fluctuations (reproducable fluctuations in the conductance versus Fermi energy or magnetic field). Another system commonly used to study mesocopic effects are very pure semiconductor heterostructures (e.g. GaAs/AlGaAs), where scattering processes are rare compared to metalls. In these highmobility systems electrons are confined to two dimensions at the interface of the two semiconducting components. Because of the large Fermi wavelength, the dimensions of such a two-dimensional electron system can be reduced even further by quantum confinement to one (quantum wire) or even zero (quantum dot) dimensions. Nowadays, heterostructure semiconductors are increasingly used in telecommunication technology and can be found in mobile phones, CD-players, bar-code reader etc.

Central topic of this thesis are fluctuation phenomena in the electrical current of high-mobility semiconductors. Such time dependent fluctuations of the current around its mean value occur due to the discrete nature of the electron charge and are called shot noise. They are present even at zero temperature. In contrast to classical music, where noise is most often a disturbance, the noise in the electrical current contains additional information on how the electrons move in a conductor. This information is not available from common conductance measurements.

For example, the statistics or the charge of the particles involved in transport can be probed by shot noise. Furthermore, electrical noise has become an alternative and very accurate method to determine the temperature of electrons in a solid. Thus, measuring the noise in the electrical current is a very powerful tool in mesoscopic physics. It will certainly play an important role in the future, too, for example within the field of quantum computing in order to probe the correlations caused by entanglement.

This thesis is organized in the following way. The second chapter gives an introduction into some basic concepts of electrical transport and noise in mesoscopic systems. Furthermore, I briefly review the properties of twodimensional electron gases. The third and fourth chapter describe the processing of heterostructure devices (chap. 3) as well as the technique to detect low-frequency noise (chap. 4). In the following chapters (chap. 5-8) the main results on fluctuation phenomena in low dimensional conductors of this thesis are presented:

Chapter 5: In the fifty's of the last century a new field, nowadays called quantum statistics, was invoked by a fundamental experiment of Hanbury Brown and Twiss (HBT). The statistics of inherent indistinguishable quantum particles is different from that of classical particles which always can be distinguished by their unique classical trajectories. As is well known, there are two different kinds of quantum particles, Bosons and Fermions, which differ in the symmetry of the wave function upon interchange of two particles. HBT explored the statistics of a thermal photon field which is made out of Bosons performing an intensity correlation experiment. In this chapter we present an analogous experiment carried out with electrons which are Fermions.

Chapter 6: The amount of shot noise in mesoscopic conductors is not arbitrary but equals various so called universal values for different systems. Here, 'universal' means that the noise level is insensitive to microscopic properties of the device. Cavities of micrometer dimensions in which electrons scatter randomly are one example of a system where the shot noise is believed to be universal. Here, we present the first experimental confirmation of this theoretical prediction for the shot noise in so called open chaotic cavities.

Chapter 7: Shot noise was first discovered in *classical* systems, namely in vacuum tubes by W. Schottky in 1918. The detailed investigation of shot noise in nano-conductores started only during the last ten years ago and since than has provided a tremendous amount of new information about charge transport. Surprisingly, the mathematical expressions for shot noise in classical systems like vacuum tubes and in coherent (mesoscopic) conductors are very similar which leads to the question about differences and similarities in the origin of shot noise in classical and quantum mechanical systems. In this chapter I discuss an experiment which clearly shows that the shot noise present in mesoscopic devices is a purely quantum mechanical effect, which disappears in the case that electronic motion is governed by laws of classical mechanics alone.

Chapter 8: In this final chapter I investigate the crossover of shot noise from a single scatterer to the limit of a large number of scatterers in series. Experimentally, each single scatterer can be modeled as a quantum point contact. Whereas for one scatterer shot noise is highly sensitive to the probability for transmission through the contact it reaches the same universal value for an infinite number of scatters independently of the transmission

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probability. Theoretically, this problem has been considered before for a series of planar tunnel junctions. In our case however, the system does not consist of a one-dimensional array of barriers, but cavities are formed in between the contacts. We therefore present a theoretical model which includes the additional cavity noise contribution to the partition noise of the contacts, and compare it to our experimental results.

Chapter 2

Mesoscopic physics

2.1 The scattering approach to transport

In classical electron transport theory (Drude model [1]) the conductivity σ of an electrical conductor follows from the balance between acceleration of the charge carriers due to an external electric field and inelastic scattering from the environment. The basic assumption behind this description is that scattering processes at different locations occur *incoherently*. Thus, for a large homogeneous conductor the conductance G = V/I, being the experimentally measured quantity, is related to the microscopic conductivity σ by

$$G = (W/L) \cdot \sigma \tag{2.1}$$

with L the length and W the width (two dimensions) or the cross-section (three dimensions) of the conductor. This scaling property of the conductance holds provided the sample size L is much larger than the mean free path l and the coherence length l_{ϕ} (the length after the phase memory of the electrons is lost). Otherwise, a local conductivity σ cannot be defined and the relevant physical quantity is the conductance G itself.¹ This is the case in mesoscopic conductors where classical concepts must be modified in light of quantum mechanics.

A well established theoretical concept - the scattering approach, also referred to as the Landauer-Büttiker formalism - describes transport in mesoscopic systems relating the conductance G of a device to the quantum mechanical transmission probability of propagating modes (quantum channels) [2]. In the following a coherent conductor ideally connected by leads² to a left and right reservoir is considered [Fig. 2.1]. Scattering within the

¹The same also holds for other physical quantities such as the specific heat.

 $^{^2 {\}rm Ideal}$ leads are *ballistic* conductors in which electrons scatter only elastically from the boundaries and backscattering is absent.





Figure 2.1: Two-terminal device coupled via two ideal leads (waveguides) to reservoirs on the left and right side.

conductor is purely elastic while all inelastic processes occur in the reservoirs. In this manner the conductor is treated as a quantum mechanical object, whereas the reservoirs are semiclassically described as a degenerate Fermi gas. The distribution functions in the reservoirs are defined via their temperatures $\theta_{L,R}$ and their chemical potentials $\mu_{L,R}$:

$$f_{\alpha}(E) = \left[\exp\left(\frac{E - \mu_{\alpha}}{k_B \theta_{\alpha}}\right) + 1 \right]^{-1}, \qquad \alpha = L, R.$$
 (2.2)

Due to the transversal confinement in the leads the energy spectrum of the electrons is quantized and so called subbands (one-dimensional channels) form. The eigenstates of these subbands, which are also denoted as modes, are

$$\psi_n^{\pm} \propto e^{\pm ik_n x} \chi_n(y, z), \qquad (2.3)$$

where n = 1, ..., N denotes the subband index and the plus (minus) sign corresponds to a right (left) moving state. The transverse wave functions $\chi_n(y, z)$ solve the time-independent Schrödinger equation in a confining potential V(y, z):

$$\left[-\frac{\hbar^2}{2m}\left(\frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right) + V(y,z)\right]\chi_n(y,z) = E_n\chi_n(y,z).$$
 (2.4)

The dispersion relation $E_n(k_n)$ of an electron in the *n*-th subband equals

$$E_n(k_n) = E_n + \hbar^2 k_n^2 / 2m^*.$$
(2.5)

A general state incoming to the scattering region S in Fig. 2.1 is a superposition of all incoming modes from the left and right reservoir. The amplitudes of these modes form a 2N vector (a_L, a_R) with N the number

2.1 The scattering approach to transport

of subbands. In the same way another 2N vector (b_L, b_R) describes an outgoing state. The various amplitudes of incoming and outgoing modes are related via the scattering matrix s:³

$$\begin{pmatrix} b_L \\ b_R \end{pmatrix} = s \begin{pmatrix} a_L \\ a_R \end{pmatrix}, \qquad s = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \equiv \begin{pmatrix} \mathsf{r} & \mathsf{t}' \\ \mathsf{t} & \mathsf{r}' \end{pmatrix}. \tag{2.6}$$

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r and t (r' and t') are the $N \times N$ reflection and transmission matrices with elements t_{nm} , r_{nm} for modes from the left (right). In case of a voltage difference V applied between the reservoirs the states in the left and right reservoirs are filled up to the energy $\mu_L = E_F + eV$ and $\mu_R = E_F$, respectively, with E_F the Fermi energy. Thus, only states within the interval $E_F < E < E_F + eV$ contribute to the net current. The *n*-th right moving mode carries a current $e \int_{E_F}^{E_F+eV} dE v_n \rho_n$ with $v_n = (1/\hbar) \cdot (dE_n/dk)$ its group velocity and $\rho_n = 1/\pi \cdot (dE_n/dk)^{-1}$ the density of states of a one-dimensional subband. The product of v_n and ρ_n is energy independent and is the same for all modes. Hence, the total current is equipartitioned among the modes in the leads each carrying the universal current amount of 2e/h per unit energy.⁴ Thereby, a fraction $(1/N) \sum_n |t_{nm}|^2$ of all incoming modes is transmitted to the right, so that the net current is given by

$$I = \frac{2e}{h} \sum_{n,m=1}^{N} \int_{E_F}^{E_F + eV} dE \, |t_{nm}|^2.$$
(2.7)

From G = I/V we obtain the Landauer formula for the 2-terminal linear response conductance (i.e. $eV \rightarrow 0$)

$$G = \frac{2e^2}{h} \sum_{n,m=1}^{N} |t_{nm}|^2 = \frac{2e^2}{h} \operatorname{Tr} \mathsf{tt}^{\dagger}$$
(2.8)

relating the conductance G to the transmission amplitudes t_{nm} from mode m to mode n. The multi-terminal generalization of Eq. (2.8) is due to Büttiker [3]. Assuming that a voltage V_{β} is applied to the reservoir β of an arbitrary device the average current through lead α at arbitrary temperature is given by [3]

$$I_{\alpha} = \sum_{\beta} G_{\alpha\beta} V_{\beta} \quad \text{with the conductance matrix}$$
(2.9)

$$G_{\alpha\beta} = \frac{2e^2}{h} \int dE \left(-\frac{\partial f}{\partial E}\right) [N_{\alpha}\delta_{\alpha\beta} - T_{\alpha\beta}].$$
 (2.10)

³In the second quantization approach incident states are described by creation operators \hat{a}^{\dagger} and annihilation operators \hat{a} , while outgoing states are described by creation operators \hat{b}^{\dagger} and annihilation operators \hat{b} .

 $^{^4\}mathrm{The}$ factor 2 accounts for spin degenerated electrons.





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Figure 2.2: (a) Measured conductance of a ballistic quantum point contact showing quantized conductance steps of $2e^2/h$. (b) The thermopower S of a quantum point contact depends on the derivative of the conductance with respect to energy leading to an oscillating behaviour of the measured thermal voltage $\Delta V_{th} = S \cdot \Delta \theta$. According to theory the peak heights are proportional to $(n + 1/2)^{-1}$ with n the subband index, what is also experimentally observed (inset).

 $T_{\alpha\beta} \equiv \text{Tr}(\mathbf{s}_{\alpha\beta}^{\dagger}\mathbf{s}_{\alpha\beta})$ is the total probability for transmission from probe β into lead α .

If no backscattering takes place inside the conductor (ballistic transport) all modes have unit transmission probability, so that in Eq. (2.8) the sum $\sum_{n,m}^{N} |t_{nm}|^2$ equals an integer number N. In this case the Landauer formula simplifies to $G = 2e^2/h \cdot N$. Such a quantized conductance in units of $G_0 \equiv 2e^2/h \simeq (12.9 \text{ k}\Omega)^{-1}$ is experimentally observed in quantum point contacts (QPC) [4, 5], which are narrow constrictions in width comparable to the Fermi wavelength [see sect. 2.2.2]. Increasing the width of the constriction the number of modes N in the point contact increases, manifesting itself in a staircase like conductance [Fig. 2.2(a)].

Like the electrical conductance thermo-electrical properties such as the thermopower $S \equiv (\Delta V/\Delta \theta)_{I=0}$, the Peltier coefficient $\Pi \equiv (Q/I)_{\Delta \theta=0}$ or the thermal conductance $\kappa \equiv -(Q/\Delta \theta)_{I=0}$ all exhibit quantum size effects, too [6, 7]. Here Q denotes the heat flow. In contrast to the conductance G, the thermopower $S \propto G^{-1}\partial G/\partial E$ probes the energy dependence of the quantum mechanical scattering processes at the Fermi energy. As a consequence of its proportionality to $\partial G/\partial E$ an oscillating thermal voltage $\Delta V_{th} = S \cdot \Delta \theta$ is observed [Fig. 2.2(b)].

2.2 The two-dimensional electron gas

2.2 The two-dimensional electron gas

The electrons of bulk metals and semiconductors are generally free to move in all three spatial directions. The energy spectrum for a free particle with effective mass m^* is

$$E(\mathbf{k}) = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2 + k_z^2)$$
(2.11)

with $k_{x,y,z}$ the components of the wave vector **k**. If the Fermi wave length $\lambda_F \equiv 2\pi/k_F$ is of the same order as the spatial width of the confining potential in a certain direction, the quantum nature of the electrons becomes significant. In that case, the energy spectrum for this direction is quantized with different subband energies E_n , and the dimensionality of the system becomes reduced. An example for a naturally occuring material showing quasi-two-dimensional behaviour is graphite, where the resistance along the sheets is much lower than between them. Other such examples are conducting polymer sheets or electrons on the surface of liquid helium. In this section, some properties of two-dimensional electron systems in semiconducting devices where the electrons are confined in z-direction at the interface of two III-V-compound semiconductors⁵ will be discussed. The

$$E_n(\mathbf{k}) = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2) + E_n, \qquad n = 1, 2, 3...$$
(2.12)

At very low temperatures and appropriate doping only the first subband E_1 is occupied so that the electron system is really two dimensional. The electrons are free to move within the xy-plane with metallic like conduction properties. The dimensionality can be even shrunken further to one dimension (quantum wires, quantum point contacts) or to zero dimension (quantum dots)⁶ by etching or electrostatic confinement [see sect. 2.2.2].

The first system where the physical properties of two dimensional electron gases (2 DEG) were studied, including the discovery of the quantum Hall effect [8] [sect. 2.2.1], is the metal-oxide-semiconductor (MOS) structure. In such a device the potential at the surface of a bulk semiconductor (typically silicon) is changed with a metal electrode on top to form either an accumulation or an inversion layer. In that way electrons or holes are trapped in a potential well forming a two-dimensional system. Such de-

 $^{^5\}mathrm{A}$ III-V-compound consists of one element of the third group and another one of the fifth group in the periodic table.

 $^{^6 \}rm Natural versions of one-dimensional systems are for example polymer chains or nano-tubes. Quantum dots can be regarded to some extend as artifical atoms.$





Figure 2.3: Different layers of a GaAs/Al_xGa_{1-x}As-heterostructure used in this thesis (top). Due to the different bandgap of GaAs and Al_xGa_{1-x}As a triangular potential well is formed on the GaAs side of the interface. The conduction band diagram as well as the carrier concentration are calculated solving the 1D-Poisson and the Schrödinger equation selfconsistently [9] [see also www.nd.edu/~gsnider/]. At low enough temperatures ($k_B\theta \ll E_F$) only the energy E_1 of the first subbband lies below the Fermi energy E_F , so that the system is only 2 dimensional. The spacer layer of 30 nm undoped AlGaAs serves to increase the electron mobility, because in this manner, scattering of the electrons from the charged impurity states (donors) can be prevented (modulation doping).

2.2	The	two-dime	ensional	electron	gas
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system	dimension	DOS $\rho(E)$	natural version
-	3	$\sim \sqrt{E}$	bulk metal
quantum well	2	$m^*/\pi\hbar^2$	graphine sheet
quantum wire	1	$\sim 1/\sqrt{E}$	nanotube
quantum dot	0	discrete	atom

Table 2.1: A system is called *n*-dimensional, if the energy spectrum is quantized in n spatial directions. 'DOS' means 'density of states'.

vices have also found a large application in industry as MOS-field effect transistors.

The physical properties of the 2 DEG in the MOS-structure are mainly determined by the roughness at the interface between the mono-cristalline semiconductor (Si) and the amorphouse oxide (SiO₂) which limits the achievable mobility $\mu ~(\equiv \vec{v}/\vec{E})$ of electrons and holes. A much higher mobility 2 DEG can be formed by burying the interface within a nearly perfect single cristall. Advanced growth techniques like molecular beam epitaxy [see chap. 3] enable semiconductors with a very low defect concentration to be grown one monolayer at a time and abrupt interfaces to be formed between semiconductors of different band gaps.

The most commonly used heterostructures are the lattice-matched GaAs/ $Al_x Ga_{1-x} As$ -compounds, with the Al mole fraction $x \simeq 0.3$. Due to the different band gap of GaAs (1.424 eV) and $Al_xGa_{1-x}As$ (1.424 eV + $x \cdot 1.25 eV$) the band diagrams of the conduction and valence band show dicontinuities. The discontinuity of the conduction band $\Delta E_C = \chi_{GaAs} - \chi_{AlGaAs}$ is given by the difference between the electron affinity χ of the two materials [10]. Fig. 2.3 shows the different layers of a typical $GaAs/Al_xGa_{1-x}As$ -heterostructure. While at the surface the Fermi niveau E_F is located 0.6 eV below the conduction band, it lies close to the valence band deep inside the bulk GaAs due to the slight intrinsic p-doping [11]. Adjusting the donor concentration in the wide band gap material $(Al_rGa_{1-r}As)$ and the thickness of the undoped spacer layer, the conduction band E_c lies below the Fermi energy E_F at the GaAs/Al_xGa_{1-x}As-interface. Electrons now accumulate in this triangular shaped potential well (inversion layer). Due to confinement the energy levels in the well are quantized. For the heterostructure shown in Fig. 2.3 only the first subband energy E_1 is smaller than the Fermi energy E_F at 4.2 K.

Two-dimensional electron gases do have several properties desirable for studying mesoscopic effects. First of all, the electron mobility is very high





Figure 2.4: Mobility (μ) of electrons in a two-dimensional electron gas in modulation-doped GaAs/AlGaAs heterostructures as a function of temperature. The highest achieved mobility of $20 \cdot 10^6$ cm²/Vs corresponds to a mean free path of the electrons of roughly 200 μ m! In metallic structures the mean free path is of the order 10 nm. The arrows indicate the mobility of 3 different heterostructures used in this thesis.

compared to bulk GaAs or metals [Fig. 2.4]. This is due to the low impurity concentration and also because of the small effective mass of the electrons $(m^* = 0.067 \, m_e)$. Furthermore, the electron density is low (compared to a metal) and can be easily varied by means of an electrical field perpendicular to the layers [see sect. 2.2.2]. In addition the low carrier density results in a large Fermi wave length. For a typical heterostructure with an electron density of $n_e \simeq 3 \cdot 10^{11} \, \mathrm{cm}^{-2} \, (E_F \simeq 11 \, \mathrm{meV})$ the Fermi wave length $\lambda_F \equiv \sqrt{2\pi/n_e} \simeq 50 \, \mathrm{nm}$.

2.2.1 Magnetotransport phenomena

The quantized Hall effect (QHE) observed in a strong magnetic field applied perpendicular to the electron motion is one of the most remarkable phenomena exhibited by a 2 DEG [8]. The Hall resistance $R_H \equiv R_{xy}$, which classically equals $B/n_e e$, exhibits precisely quantized plateaus at integer (and fractional) multiples of $h/e^2 \simeq 25.8 \text{ k}\Omega$ [Fig. 2.5]. The QHE is due to the formation of highly degenerated quantized energy levels (Landau levels) in the energy independent 2D density of states:

$$E_{\nu} = (\nu - 1/2) \hbar \omega_c \qquad \nu = 1, 2, 3, \dots, \tag{2.13}$$

with $\omega_c = eB/m^*$ the cyclotron frequency. The filling factor ν which appears in Eq. (2.13) denotes how many of these Landau levels are occupied. Simultaneously to the quantized Hall resistance, the longitudinal 4pt-resistance $R_L \equiv R_{xx}$ vanishes (if $\mu B \gg 1$ and $\hbar \omega_c \gg k_B \theta$). Both effects, the quantized Hall resistance and the 'zero' longitudinal resistance can be understood in terms of one-dimensional magneto-electric subbands





Figure 2.5: Longitudinal resistance R_{xx} and Hall resistance R_{xy} measured in a very high mobility 2 DEG ($\mu = 6 \cdot 10^6 \text{ cm}^2/\text{Vs}$) at 270 mK.

carrying current along the boundaries of the device $(edge \ states)^7$ [12, 13] [see inset of Fig. 2.5]. The formation of these edge states results from the bending of the Landau levels due to the confinement at the boundaries of the conductor. The intersection of the Landau levels with the Fermi level leads to delocalized states, which carry the current in *opposite* directions on *opposite* sides of the sample. The number of edge states is given by the filling factor ν . In the bulk region of the conductor all states are localized due to potential fluctuations in the interior of the sample. Because of the spatial

 $^{^7{\}rm Edge}$ states are the quantum-mechanical analog of skipping orbits of electrons undergoing repeated specular reflections at the boundary of the device.

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Figure 2.6: (a) Fractional quantization of the the longitudinal Hall resistance in the integer quantum Hall regime for filling factors $\nu = 2, 4, 6, 8$ and 10. The numbers at the plateaus denote the resistance of the plateau according $(1/\nu - 1/M)$ with $M = t\nu$ the number of transmitted modes. The curves are horizontally shifted for clarity. (b) Hall bar with split gate structure to backscatter one edge state while the other is transmitted.

separation of states carrying current in one direction from those carrying current in the opposite direction, backscattering is completely suppressed explaining why the voltage drop between two voltage contacts along the same side is zero ($R_{xx} = 0$). The distinction between R_{xx} and R_{xy} is topological: Each edge state carries a current amount of e/h per unit energy and the potential drop between any two voltage contacts on *opposite* sides equals $(\mu_L - \mu_R)/e$, so that the Hall resistance follows as

$$R_{xy} = R_{14,35} = \frac{(\mu_L - \mu_R)/e}{I} = \frac{(\mu_L - \mu_R)/e}{\nu(e/h) \cdot (\mu_L - \mu_R)} = \frac{1}{\nu} \frac{h}{e^2}.$$
 (2.14)

This simple picture of the quantized Hall effect in terms of single-electron states holds for integer filling factors ν (i.e. for the integer quantum Hall regime). The theory for the quantum features observed at fractional $\nu's$ in high mobility devices ($\mu > 10^6 \text{ cm}^2/\text{Vs}$) invokes many-electron effects, too [for a short review, see Ref. [14]]. This fractional quantum Hall effect





Figure 2.7: Shubnikov-de Haas oscillations used to determine the carrier density n_e and the mobility μ of a 2 DEG. At $B \sim 0.3$ Tesla the additional Zeeman splitting $\Delta E = \pm \frac{1}{2}g^*\mu_B B$ ($g^* = -0.44$ in GaAs) of the Landau Levels becomes apparent.

is the result of the highly correlated motion of many electrons in 2D exposed to a magnetic field. Actually, the fractional states can be described as integer quantum Hall states of so called composite fermions, which are quasiparticles made out of one electron attached with two magnetic flux quanta $\phi_0 \equiv h/e$, moving in an effective magnetic field $B_{eff} = B - B_{\nu=1/2}$. Thus, the state at $\nu = 1/2$ where one Landau Level is half filled is of special interest, since it can be regarded as a Fermi sea of composite fermions at $B_{eff} = 0$, i.e. in the apparent absence of a magnetic field.

Besides, 'fractional' quantization of the longitudinal resistance can also be observed in the integer quantum Hall regime, namely, when backscattering is artifically introduced with the help of a split gate [see sect. 2.2.2] across the Hall bar [Fig. 2.6(b)]. From a standard exercise of the Landauer-Büttiker formalism, the longitudinal resistance follows as [13]

$$R_{xx} = \frac{h}{e^2} \left(\frac{1}{\nu} - \frac{1}{M} \right) \tag{2.15}$$

with $M = t \nu$ the number of modes transmitted at the point contact. Experimental results illustrating Eq. (2.15) are given in Fig. 2.6(a).

From an experimental point of view the magnetoresistance data like the oscillations in R_{xx} (Shubnikov-de Haas oscillations, see Fig. 2.7) and the low-field data of the Hall-resistance R_H are commonly used to characterize the two-dimensional electron gas, i.e. to determine the carrier density n_e and the electron mobility μ .

2.2.2 Gated nanostructures

Two-dimensional electron gases can easily be given an arbitrary shape using lithographic techniques. This is achieved either by etching a portion of the





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Figure 2.8: (a) Two-dimensional capacitor at the edge of a 2 DEG. The gate and the 2 DEG are assumed to lie in the same plane. n_e is the electron density of the 2 DEG far away from the boundary. (b) Too long point contacts or impurities nearby the contact destroy the proper conductance quantization. The superimposed oscillations are due to interference of electrons which undergoe multiple reflexions at the entrance and the exit of the contact.

2 DEG (permantly) or by using metallic top gates (reversal). Applying a negative voltage $-V_G$ to the gates, electrons underneath are repelled leaving a depleted strip behind [Fig. 2.8(a)]. The width of this strip is given by the depletion length $l_d = 2\epsilon\epsilon_0 V_G/\pi n_e e$. For $\epsilon_{GaAs} = 13.1$, $V_G = -2$ V and $n_e = 2.7 \cdot 10^{15} \text{ m}^{-2}$, which are typical parameters, the depletion length l is of the order 350 nm. The spatial carrier density n(x) for $x > l_d/2$ can be calculated solving the Laplace equation $\Delta \phi(\mathbf{r}) = 0$ in the half-space z < 0 ($\epsilon \gg 1$) [15]:

$$n(x) = n_e \sqrt{\frac{2x - l_d}{2x + l_d}}.$$
(2.16)

 n_e denotes the electron density of the 2 DEG far away from the boundary. Deep inside the 2 DEG the external potential is perfectly screened and the electron density is homogeneous.

One example of a gated nanostructure is a QPC defined by a pair of metallic split gates [Fig. 2.9(a) and Fig. 8.4(b)], forming a narrow constriction in the 2 DEG. In a simple approximation such a constriction can be modeled as a square well potential of length L and width W. A more realistic description is achieved by a saddle-point shaped potential [Fig. 2.9(b)] [16]:

$$V(x,y) = V_0 + \frac{1}{2}m^*(\omega_y^2 y^2 - \omega_x^2 x^2).$$
(2.17)

Due to the lateral confinement a series of 1D subbands $(N = W/(\lambda_F/2))$





Figure 2.9: (a) Metallic split gate structure on top of a 2 DEG [see also Fig. 8.4(b)]. When a negative voltage is applied to the metallic gates, the lithographic pattern is electrostatically transferred into the 2 DEG forming a narrow constriction with quantized conductance. (b) Saddle-point potential modeling a quantum point contact with $\hbar\omega_x = 1$ meV and $\hbar\omega_y = 2$ meV.

forms each contributing to the conductance by $2e^2/h$. For the saddle-point potential (2.17) the transmission eigenvalues at the Fermi energy are [16]

$$T_n = [1 + \exp(-2\pi E_n/\hbar\omega_x)]^{-1}$$
(2.18)

with $E_n \equiv E_F - V_0 - (n - 1/2) \hbar \omega_y$. Thus, in ballistic constrictions, where the length of the constriction is much smaller than the mean free path, the conductance G of a QPC goes down in discrete steps of $2e^2/h$ as the width W of the constriction is decreased by increasing the applied negative gate voltage as shown Fig. 2.2(a). However, the quantization is not as precise as for the quantum Hall effect. In order to observe conductance quantization the constriction must be adiabatically coupled to the reservoirs, which means that there is no intersubband mixing. The criterion is that the width of the constriction changes smoothly: $dW(x)/dx \leq N^{-1}(x)$. An optimal length for the occurence of quantized conductance plateaus is $L_{opt} \simeq 0.4 \sqrt{W \lambda_F}$ [17]. If $L < L_{opt}$ the plateaus show a finite slope, whereas in the case of $L > L_{opt}$ oscillations are superimposed on the plateaus, which are due to multiple reflexions of the electrons at the entrance and the exit of the contact leading to interference effects [Fig. 2.8(b)]. Especially for noise measurements on point contacts [see chap. 6-8] the design of the contacts is crucial, since any non-linearities in the IV-characteristics of the contacts makes noise measurements very difficult to be performed [sect. 4.2.2].

2 Mesoscopic physics

2.3 Current fluctuations

Electronic current noise are dynamical fluctuations $\Delta I(t) \equiv I(t) - \langle I \rangle$ in the electrical current I(t) around its time averaged mean value $\langle I \rangle$. While the conductance G describes the time-averaged current, noise can provide additional information about electronic transport properties which are not contained in the conductance itself. A detailed description of electrical noise in the time domain is given by the correlation function

$$C_{I,\alpha\beta}(t) \equiv \langle \Delta I_{\alpha}(t+t')\Delta I_{\beta}(t') \rangle \tag{2.19}$$

with $\Delta I_{\alpha,\beta}$ the current fluctuations at probe α and β of an arbitrary device. The brackets denote an average over an ensemble of identical physical systems or (ergodicity assumed) an average over the initial time t'. Equivalently, electrical noise might be presented in the frequency domain by the power spectral density [18]

$$S_{I,\alpha\beta}(\nu) = 2 \int_{-\infty}^{\infty} dt \, e^{i \, 2\pi\nu t} \cdot C_{I,\alpha\beta}(t).$$
(2.20)

Intrinsic current noise is due to fluctuations in the occupation number of states which are caused by (i) thermally activated fluctuations (thermal noise) and (ii) by the randomness inherent in quantum-mechanical transport (shot noise). In fact, the latter noise source is a direct consequence of the quantization of charge.

2.3.1 Thermal noise

At temperatures $\theta \neq 0$ thermal noise is always present in any conductor. In the following we give a classical derivation for thermal noise considering a short-circuited classical resistor R of length L which is assumed to be in thermal equilibrium [see Ref. [19]]. The average kinetic energy of an electron moving in the x-direction is $m^* \langle v_x^2 \rangle / 2 = k_B \theta / 2$. The current pulse i which is attributed to the free propagation of a single electron over the mean free path l during the collision time τ is

$$i = \frac{l}{L} \left(\frac{e}{\tau}\right) = \frac{e}{L} v_x. \tag{2.21}$$

Because $\langle i \rangle = 0$ the variance in the current pulses of one electron over a large number of collisions follows then as

$$\langle \Delta i^2 \rangle \equiv \langle i^2 \rangle - \langle i \rangle^2 = \frac{e^2 \langle v_x^2 \rangle}{L^2} = \frac{e^2 k_B \theta}{L^2 m^*}.$$
 (2.22)

2.3 Current fluctuations

For N electrons the variance of the total current is $\langle \Delta I^2 \rangle = N \langle \Delta i^2 \rangle$, where $N = L^2/e\mu R$ with the electron mobility $\mu = em^*/\tau$. Thus, it follows

$$\langle \Delta I^2 \rangle \equiv C_I(t=0) = N \cdot \frac{e^2 k_B \theta}{L^2 m^*} = \frac{k_B \theta}{R} \frac{1}{\tau}.$$
 (2.23)

If the single-charge events are uncorrelated in time the correlation function $C_I(t)$ is decaying exponentially [20, 21]:

$$C_I(t) = C_I(0) e^{-|t|/\tau}.$$
(2.24)

Integration according to Eq. (2.20) yields a current spectral density of

$$S_{I}(\nu) = 2k_{B}\theta G \frac{1}{\tau} \cdot \int_{-\infty}^{\infty} dt \, e^{i\,2\pi\nu t} \, e^{-|t|/\tau}$$

$$= 4k_{B}\theta G \frac{1}{\tau} \cdot \int_{0}^{\infty} dt \, \cos(2\pi\nu t) \, e^{-|t|/\tau}$$

$$= 4k_{B}\theta G \cdot \frac{1}{1+(2\pi\nu\tau)^{2}}$$

$$\simeq 4k_{B}\theta G \qquad (2.25)$$

for $\nu \ll \tau^{-1}$. Eq. (2.25) is known as the Johnson-Nyquist relation [22, 23] and provides an example for the fluctuation-dissipation theorem⁸. Thus, the investigation of thermal noise does not provide more information than the investigation of the AC conductance. At very high frequencies when $h\nu \ge k_B \theta$ vacuum fluctuations contribute to the equilibrium fluctuations, too. In this case Eq. (2.25) has to be changed by replacing $k_B \theta$ by the expression $(h\nu/2) \coth(h\nu/2k_B\theta)$, which equals the classical expression for $h\nu \ll k_B \theta$ [24]. Consequently the noise is no longer frequency independent ('white') for $\nu > k_B \theta/h$, but increases linearly with frequency.

2.3.2 Shot noise

Shot noise in an electrical conductor is a non-equilibrium phenomenon which is due to the randomness in the transmission of discrete charge quanta q from source to drain [25]. The shot noise of a single barrier with transmission probability T can be understood simply from classical statistical arguments. Assume there are n charge quanta q incident on the barrier per unit time τ . The distribution of the number of transmitted particles n_T is then binomial:

$$p_n(n_T) = \binom{n}{n_T} T^{n_T} (1 - T)^{n - n_T}.$$
 (2.26)

⁸The fluctuation dissipation theorem states that the linear response on a an external force equals $1/k_B\theta$ times the variance of the conjugated variable while the force is absent.

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Figure 2.10: To get a feeling for shot noise one could think of the granular sound one hears while listening to a droping tap. As long as the water flow is not to large, the granularity of the water made out of small droplets is still perceptible. In macroscopic electrical conductors shot noise is typically absent. This is similar to the case when somebody empties a whole bucket of water. In this case the granularity of the water is completely lost, too, and one hears just an averaged single sound [29].

The time averaged number of transmitted particles $\langle n_T \rangle$ equals nT, while the variance is given by [see app. A]

$$\langle \Delta n_T^2 \rangle \equiv \langle n_T^2 \rangle - \langle n_T \rangle^2 = nT (1 - T) = \langle n_T \rangle (1 - T).$$
 (2.27)

With $I = qn_T/\tau$ the variance of the total current is $\langle \Delta I^2 \rangle = q \langle I \rangle / \tau \cdot (1-T)$. Using Eq. (2.24) and integration according Eq. (2.20) yields for $\nu \ll \tau^{-1}$ a frequency independent shot noise power of

$$S_I = 2q\langle I \rangle \cdot (1 - T). \tag{2.28}$$

In the limit of very low transmission probability $(T \rightarrow 0)$ the binomial distribution (2.26) can be approximated by the Poisson distribution. In this case shot noise is given by the well known Schottky formula [26]:

$$S_I = S_{Poisson} \equiv 2q\langle I \rangle. \tag{2.29}$$

If the charge would not be quantized, shot noise would be absent, i.e. $S \to 0$ for $q \to 0$. Generally, the Poissonian value $2q\langle I \rangle$ is used as a relative measure to compare any noise. Especially in mesoscopic systems [see below] correlations imposed by fermionic statistics of the electrons as well as Coulomb interaction may change shot noise from $S_{Poisson}$. This is expressed by the Fano factor F defined as the zero-temperature excess noise normalized to the Poisson noise:

$$F \equiv \frac{S_I}{2e\langle I \rangle}.$$
 (2.30)

Shot noise has been observed in classical devices as well as in mesoscopic systems. Poissonian noise is for example present in the current of vacuum

2.4 Shot noise in mesoscopic systems

tubes [27, 28] or tunnel junctions. In other macroscopic devices like macroscopic metallic wires shot noise is absent because the granularity in the charge flow is smeared out by inelastic scattering of the electrons with the environment. In recent years shot noise in mesoscopic systems has extensively been studied providing a lot of new information on charge transport, especially on the:

quantum of charge of the carriers involved in transport. The proportionality to the quantum of charge q according to the Schottky formula (2.29) in case of low transparencies has for example been employed to determine the effective charge in superconducting transport [30] or in the fractional quantum Hall regime [31, 32].

statistics, in a stream made out of identical quantum particles, or

quantum partitioning, the way charge carriers scatter and interact within a mesoscopic device.

2.4 Shot noise in mesoscopic systems

A quantum coherent theory of noise has been derived within the framework of the scattering approach [see Ref. [33]]. Similar to the conductance Eq. (2.10) the general result is a relationship between the shot noise power and the transmission matrix at the Fermi energy. For a two-terminal configuration [Fig. 2.1] it is found [34, 35] [see also app. C]

$$S_{I} = 2\frac{e^{2}}{h} \int_{0}^{\infty} dE \left\{ \left[f_{L}(1 - f_{R}) + f_{R}(1 - f_{L}) \right] \operatorname{Tr} \operatorname{tt}^{\dagger}(1 - \operatorname{tt}^{\dagger}) + \left[f_{L}(1 - f_{L}) + f_{R}(1 - f_{R}) \right] \operatorname{Tr} \operatorname{tt}^{\dagger} \operatorname{tt}^{\dagger} \right\}.$$
(2.31)

In the basis of eigen-channels of the matrix tt^{\dagger} this result can be expressed by the set of eigenvalues (transmission probabilities) T_n of tt^{\dagger} :

$$S_{I} = 2\frac{e^{2}}{h}\sum_{n=1}^{N}\int_{0}^{\infty} dE \left\{ [f_{L} - f_{R}]^{2} T_{n}(E)(1 - T_{n}(E)) + [f_{L}(1 - f_{L}) + f_{R}(1 - f_{R})] T_{n}(E) \right\}.$$
(2.32)

If the response is linear so that we can neglect the energy dependence of the transmission matrix the Fano factor is

$$F = \frac{\sum_{n} T_{n} (1 - T_{n})}{\sum_{n} T_{n}}.$$
 (2.33)





Figure 2.11: Measured Fano factor of a single QPC defined in a twodimensional electron gas [see subsect. 2.2.2] as a function of the sum of transmission eigenvalues T_n after Kumar *et al.* [36]. There is a very good agreement of the experimental data with the theoretically expected value of Eq. (2.33) and (2.34).

Provided that all transmission eigenvalues are small $(T_n \ll 1)$ the Fano factor equals 1 and the shot noise is Poissonian. Obviously, the Fano factor and the shot noise are zero for a ballistic conductor, where all T_n equal unity.

2.4.1 Few-channel quantum conductors

For a one-channel quantum conductor, e.g. a QPC where only one mode contributes to the current, the Fano factor is simply given by [34, 35]

$$F = 1 - T.$$
 (2.34)

This has also been experimentally confirmed with very high accuracy [36] [see Fig. 2.11]. At finite temperatures θ thermal noise of the quantum point contact is present, too. In the tunneling regime $(T \ll 1)$ the crossover from thermal noise at voltages $e|V| \ll k_B \theta$ to pure shot noise at voltages $e|V| \gg k_B \theta$ is given by [37]

$$S_I = S_{Poisson} \cdot \coth\left(\frac{e|V|}{2k_B\theta}\right). \tag{2.35}$$

Atomic-size metallic contacts (break junctions) are another example of a conductor with only a few modes. For these systems shot noise can be used in addition to conductance measurements in order to determine the number of channels at the contact as well as their transmission eigenvalues [38].





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Figure 2.12: Distribution of transmission eigenvalues T for (a) a diffusive metallic wire according Eq. (2.36) and (b) a symmetric chaotic cavity according Eq. (2.37).

2.4.2 Multi-channel quantum conductors

In other mesoscopic conductors where the number N of transmission eigenvalues T_n is large (multi-channel quantum conductors) the Fano factor (2.33) depends on the distribution $p(T_n)$ of the eigenvalues $T_n \in [0,1]$. Metallic diffusive wires shorter than the electron-phonon interaction length and *chaotic cavities* are two examples of such systems with high degree of freedom $(N \gg 1)$ [see insets of Fig. 2.12]. The first one is characterized by a wire length L much larger than the mean free path l so that electrons diffuse from the left reservoir through the wire to the right reservoir while they elastically scatter from randomly placed impurities. On the other hand, a chaotic cavity is specified by dimensions smaller than the mean free path lso that electrons scatter ballistically within the cavity. The cavity is coupled via two noiseless contacts to reservoirs on the left and right. Generally, the distribution p(T) of transmission eigenvalues can be calculated from random matrix theory (RMT), which deals with the statistical properties of large scattering matrices. These are chosen from an ensemble representing the symmetry of the system [for a review, see Ref. [17]]. For both systems - a metallic diffusive wire and a chaotic cavity - the distribution functions p(T) are bimodal with a peak at T = 0 and T = 1. They are given by

$$p(T)_{wire} = \frac{l}{2L} \frac{1}{T\sqrt{1-T}} \qquad \text{(diffusive wire)} \qquad (2.36)$$

$$p(T)_{cavity} = \frac{1}{\pi} \frac{1}{\sqrt{T(1-T)}} \qquad \text{(chaotic cavity)}. \qquad (2.37)$$

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F	physical system	T_n	experiments
0	ballistic conductor	1	[36, 38, 43, 44, 45]
1/4	chaotic cavity	bimodal	[46]
1/3	diffusive wire	bimodal	[47, 48, 49]
1/2	symmetric double-barrier	bimodal	[50, 51]
1	single tunnel-barrier	$\ll 1$	[51]

Table 2.2: Overview over various universal Fano factors $F \equiv S/S_{Poisson}$ observed in mesoscopic devices. The rigth column gives a selection of corresponding experimental works. Except when negative differential conduction occurs [52] the Fano factor of normal conducting systems is in the range 0 < F < 1 meaning that shot noise is partially suppressed. In normal-metal/superconductor hybrid structures shot noise is enhanced due to multiple Andreev reflection [30, 53, 54, 55, 56] and thus, the Fano factor can be larger than 1. Very recently, a Fano factor larger than 1 has also been observed in the highly correlated regime of the FQHE [57].

Both distributions, illustrated in Fig. 2.12, are universal in the sense that they are insensitive to microscopic properties of the device. Together with the general expression (2.33) the Fano factors $F = S_I/S_{Poisson}$ follow as

$$F_{wire} = 1/3 \qquad \text{(diffusive wire [39])} \qquad (2.38)$$

$$F_{cavity} = 1/4 \qquad \text{(chaotic cavity [40])}. \qquad (2.39)$$

Electron-electron interaction enhances these universal Fano factors. In case of hot electrons $F_{wire} = \sqrt{3}/4 \simeq 0.43$ [41] for a diffusive wire and $F_{cavity} = \sqrt{3}/2\pi \simeq 0.276$ for a chaotic cavity [42]. An overview over different universal Fano factors for various mesoscopic systems is given in Tab. 2.2.

The description of shot noise within the scattering approach is part of a completely phase-coherent theory. Characterizing the shot noise of an ensemble of various devices, which differ only in microscopic properties such as the arrangement of the scattering centers or the exact shape of the boundaries, it is not necessary to include the information about phases of wave functions. That is why the universal Fano factors (2.38) and (2.39) can be derived from a semiclassical analysis, too [58, 59, 60].⁹ Within the semiclassical approach transport and noise are described by single particle distribution functions $f_{\mathbf{p}}(\mathbf{r},t)$ and two-particle correlation functions $\mathcal{F}_{\mathbf{pp}'}(\mathbf{rr}',t) = \langle f_{\mathbf{p}}(\mathbf{r},t) f_{\mathbf{p}'}(\mathbf{r}',0) \rangle$ satisfing the classical Boltzmann equation

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⁹Similar to the universal conductance fluctuations away from the average conductance, it is possible to find the fluctuations of the noise power away from its ensemble average behavior [61].





Figure 2.13: (a) Shot noise of a metallic diffusive wire [from Ref. [49, 62]]. For sensitivity reasons 8 wires are measured in series with reservoirs in between. Special care had to be taken to minimize heating effects due to the large applied bias voltages $(eV \gg k_B\theta)$. The reservoirs are very thick and large to maximize the cooling power. (b) Scanning electron microscope picture of one of 8 wires between two reservoirs. [49, 62].

[see also chap. 6 and app. B]. Furthermore, this approach also allows to easily incorporate interaction effects between electrons (heating).

The 1/3-shot noise suppression (2.38) in metallic diffusive wires is experimentally well confirmed [48, 49, 62]. Experimental results are given in Fig. 2.13(a). In contrast, the experimental verification of the 1/4-shot noise predicted for chaotic cavity has been an outstanding problem [25]. It is one of the central topics of this thesis [see chap. 6].

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Chapter 3

Processing of heterostructure devices

In this chapter the preparation of the devices is described. Lateral structuring is carried out using standard optical and electron-beam lithography. In general, the preparation includes three main steps: (i) the ohmic contacts are alloyed to the 2 DEG, (ii) the 2 DEG is structured by wet-chemical etching and (iii) metallic gates are deposited, which are used to deplete the electron gas electrostatically. This third step is typically split into two, one for the fine gates (e.g. QPCs) and another one for the connection to the bonding pads.

MBE-growth

The starting material for the devices investigated in this thesis are AlGaAs/GaAs-heterostructures. The atomic structure of GaAs is shown in Fig. 3.1. In contrast to silicon, GaAs has a direct bandgap and is therefore very important for optical applications, too (semiconducting laser diodes). Furthermore, the material is widely used in high-frequency and low noise transistors (high-mobility field effect transistor). A sequence of the layers in a typical heterostructure is given in Fig. 2.3. In order to create semiconducting alloy structures with extremly sharp interfaces between one type of alloy



Figure 3.1: Atomic structure of GaAs (Zinkblende). In $Al_x Ga_{1-x} As$, x denotes the fraction of Ga-atoms replaced by Al-atoms. Typically, the difference between the lattice constant a for the two materials is of the order 0.1 %





Figure 3.2: (a) Schematics of a molecular-beam epitaxy (MBE) chamber. Molecular or atomic beams of the constituents are generated from different effusion cells and travel withouth scattering to a substrate where they combine to form an epitaxial film. (b) Typical growth rathes are one monolayer per second. In order to increase the mobility of the atoms on the surface the substrate is typically heated up to 600 $^{\circ}$ C. During the MBE process the growth can be monitored in situ by reflection high energy electron diffraction (RHEED).

and the next (i.e. AlGaAs and GaAs) atomic layers are grown monolayer by monolayer in the process of molecular beam epitaxy (MBE) [63]. The different heterostructures used in this thesis are all MBE grown. This technique provides a way to produce high quality materials with a low number of defects, so that the mobility of electrons and holes can be extremely high. A schematic drawing of a MBE chamber is shown in Fig. 3.2(a): molecular beams of the constituent elements (Ga, Al, As, Si, Ge, ...) travel within an ultra high vacuum chamber ($p < 10^{-11}$ mbar) from different effusion cells to a substrate, where the atoms combine to form an epitaxial film. The elements Si and Ge are thereby used as dopants. During epitaxial growth the atoms on the clean surface are free to move around until they find their correct lattice position to form chemical bonds [Fig. 3.2(b)]. There are other faster and more economic growth techniques than MBE, which also do not require ultra high vacuum techniques. Nevertheless, the high mobility heterostructures for research purposes can only be fabricated by MBE.

Lithography

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Optical as well as electron beam lithography are based on the same principle: an organic resist is coated onto the substrate and polymerized to some extent by baking in an oven or on a hot plate. When exposed to light (optical lithography) or to a focused electron beam (e-beam lithography) some materials (the positive resists) are broken into smaller organic units, that are more easily dissolved by a liquid solvent. Other (negative resists) polymerize further so that solvents subsequently remove those parts which were not exposed [see Fig. 3.3]. In optical lithography all structures are exposed in a single flash using a so called optical mask (metal film on glass) which is opaque for some parts and transparent to light for other parts. The exposure time is independent of the size of the structures. In contrast to the parallel exposure process in optical lithography, exposing with a focused electron beam is serial requiring the beam to be scanned step by step over the resist, and thus, is much more time consuming. Nevertheless, in order to fabricate nanostructures electron beam lithography is essential, because diffraction limits the smallest feature achievable with optical lithography to $\simeq 0.7 \ \mu m$ [64], whereas this limit is $\lesssim 30 \ nm$ in e-beam lithography. Furthermore, e-beam is much more flexible, since the patterns are not fixed but can be easily altered, whereas for optical lithography a completely new mask has to be created.

For the processing of heterostructure devices both lithography techniques are typically combined. Here, we defined the ohmic contacts as well as the protection mask for etching the 2 DEG by optical lithography. A negative photoresist has been used, and we were working with both 'positive' and 'negative' masks. One of these structures is shown in Fig. 3.4. In the first lithography step the areas marked as ohmic contacts are exposed.



Figure 3.3: Overview over standard lithography steps. In our case we used a negative resist for optical lithography (1a) and a positive resist (PMMA) for ebeam (1b). Further steps are development (2), metallization (3) and lift-off (4). Further details can be found for example in Ref. [65].





Figure 3.4: Structures on an optical mask used to define the ohmic contacts, the Mesa as well as the large scale gate structures. The masks were designed in a GDSII-format and commercially produced by Photronics S.A. in Neuchâtel, Switzerland.

Ohmic contacts

An ohmic contact to a semiconductor device has ideally a linear currentvoltage characteristic and a very low resistance compared to the semicondutor. When a metal and a semiconductor are put in contact to each other they usually form a Schottky barrier with the barrier height of several eV. To achieve a good low-ohmic contact, a heat treatement is required to allow the metal into the surface of the semiconductor [64, 66]. Only for some very special systems is it possible to get non-alloyed ohmic contacts for example for InAs with In as the contact metal. In general, the semiconductor has to be highly doped at the interface to the metal so that the depletion region, formed by the Schottky barrier, becomes very thin and the tunneling current through the barrier is strongly enhanced [see Fig. 3.5]. Alloying a multilayer structure of Au-Ge-Ni is the usual method for obtaining highlydopped regions in n-type GaAs or AlGaAs . Thereby gold and germanium are deposited in their eutectic mixture of 88:12 wt% providing a low melting point of about 360 °C. For contacts to p-type GaAs, Zn is used instead of Ge and Ni [64]. During the alloying process Ge diffuses into the Al-GaAs and forms a strongly doped region. The diffusion of germanium into the semiconductor is increased by Ni $[67]^1$. Gold adheases the contact to

 $^{^{1}}$ It is crucial that there is no residual optical resist in between the semiconductor surface and the eutecticum, otherwise the diffusion is blocked. That is why after develo


Figure 3.5: Band diagrams of a Schottky contact (a) and an ohmic contact (b) in equilibrium. In the first case thermionic emission above the barrier height is the principal mechanims for electrons to overcome the barrier (under forward bias). By doping the semiconductor near the surface, the depletion region caused by the Schottky barrier becomes much thinner and so tunneling through the barrier increases the current and gives a linear IV-characteristic. (c) Schematic eutectical diagram of Au and Ge. (d) left: nonalloyed contact; right: the alloyed region reaches the 2 DEG and ensures an ohmic contact.

the surface and makes it possible to bond onto the contact after annealing. The alloying takes place in an annealing oven² under a continuous flow of forming gas (90% N₂ + 10% H₂), preventing any oxidation. The continuous gas flow helps to achieve large heating and cooling rates as well. Typical parameters for alloying are 400 - 500 °C for 30 to 90 s depending on the specific heterostructure. For too long alloying times the contact resistances increase, what is also well known in literature [67]. Normally, if the contacts have a low-ohmic resistance at room temperature, they are also good at liquid helium temperature provided they are cooled down slowely. The

opment the device is exposed to an oxygen plasma for cleaning. The oxide layer created during this procedure can afterwards be removed by a dip-etch in concentrated HCl.

 $^{^2\}mathrm{AZ}$ 500 from MBE-Komponenten GmbH, Germany.





Figure 3.6: High mobility two dimensional electron gas (2DEG) contacted with Indium and mounted in a chip carrier. The 2 DEG is located 222 nm below the surface. Fractional quantum Hall effect measurements on this device performed in a van der Pauw-geometry are shown in Fig. 2.6.

contacts are sometimes altered by thermal cycling.

The procedure for achieving ohmic contacts as described above works fine for heterostructures of 'normal' mobility ($\mu \leq 10^6 \text{ Vs/cm}^2$) with a rather large electron density ($2 \cdot 10^{-15} \text{ m}^{-2}$). For extremely high mobility heterostructures, which are for example used to study the fractional quantum Hall effect [see sect. 2.2.1], the spacer layer is much thicker so that the 2 DEG is located three times deeper below the surface than in regular devices. Furthermore, the electron density n_e is lower, too. For such heterostructures ohmic contacts cannot be made out of Au-Ge-Ni, but Indium has to be used instead [Fig. 3.6]. Thereby, the Indium is soldered by hand directly onto the wafer and is afterwards annealed.



Figure 3.7: (a) Optical microscope image of a wet-chemically etched Hall bar with two voltage probes. (b) Optical microscope image of a nearly finished device with annealed ohmic contacts and gate contacts. Various alignement marks serve for further e-beam lithography steps.

Mesa definition

The parts where the two-dimensional electron gas shall be removed are wetchemically etched a few nm. The remaining areas on the wafer are called Mesa. For the large scale definition of the Mesa again optical lithography has been used [see Fig. 3.4]. For finer structuring of the 2 DEG e-beam lithography is more convenient. The resists act as a protection layer against etching. GaAs and AlGaAs are both removed by the isotropic, non selective etchant H_2SO_4 : H_2O_2 : $H_2O = 3 : 1 : 100$. This etchant provides an overcut, so that a continuous film can be deposited over the etch of the Mesa. If the ratio H_2SO_4 : H_2O_2 is 1 : 1, an undercut is formed [68].

Metallic gates

The metallic gate structures of submicron dimensions are fabricated using electron beam lithography. The e-beam writing system consists of a JEOL JSM-IC 848 scanning electron microscope with a motorized stage and a commercially available writing software [69]. As a resist poly-methylmethacrylate (PMMA), being a standard positive resist for e-beam, has been used. After development the PMMA-layer serves as a mask for selected-area metal deposition. The deposition of the metals takes place in a high-vacuum evaporation chamber, where the metal is heat evaporated either by an electron gun or resistively. In areas where the resist on the device has been removed the evaporated metal makes direct contact with the underlying surface, otherwise it coats the resist. Finally, the remaining PMMA is re-



Figure 3.8: (a) Scanning electron microscope image of a chaotic cavity [see chap. 6]. (b) Pattern defined in a lithography software from Raith GmbH⁴ used to expose the structure in (a). The focused electron beam moves along the lines, which are close enough so that the area in between them is exposed, too. The spiral arrangement of the lines helps to correct the proximity effect, i.e. that the structure is not partially over- or underexposed.

3 Processing of heterostructure devices



Figure 3.9: (a) Device mounted in a chip carrier and contacted with Al bonding wires. On the left and the right there are five ohmic contacts while the bonding wires from the top and the bottom make contacts to several gates. (b) The bonding machine can also be used to interconnect various structures on chip, e.g. to bridge over parts which are broken [62].

moved and the metal covering the resist is 'lifted off'. The gates are made out of 40 nm gold-layer with a 2 nm thick titanium layer underneath, that provides good adheasion to the semiconductor surface.

Bonding

In order to measure a finished deviced it is mounted in a chip carrier and contacted with the aid of an ultrasonic bonding-machine which solderes 50 μ m thin aluminium-wires from the pad on the chip carrier to the contact pad on the device. A bonded device ready for measuring is shown in Fig. 3.9(a). Furthermore, the bonding wires can also be used for interconnection on the chip itself as shown in Fig. 3.9(b).

For the inspection of the finished devices we used a scanning electron microscope from *Philips* (XL30 FEG).

Chapter 4

Measurement techniques

4.1 Low temperatures and filtering

Mesoscopic effects like the quantization of the conductance in ballistic point contacts typically occur on an energy scale of one meV or smaller. In order to measure these effects the experiments are usually performed at temperatures below one Kelvin to avoid thermal smearing. Such low temperatures are achieved with the help of cryogenic liquids with a very low boiling point. Two different types of cryostats, a ³He- and a ⁴He-system, have been used in this thesis with bath temperatures of 270 mK ($\sim 23 \ \mu eV$) and 1.7 K (~ 146 μeV), respectively. The principle of these cryostats relies on isentropic cooling with liquid helium boiling under reduced pressure. Since the vapour pressure of liquid ³He at a given temperature is higher than the one of ⁴He at the same temperature, lower temperatures can be achieved in a ³He- than in a ⁴He-system. In the ³He-cryostat the ³He gas, which is stored in a closed system, condensates at the 'one-Kelvin pot' cooled down below 2 K. Further lowering of the temperature is then achieved pumping away the ³He-vapour using an adsorption pump with a temperature sensitive adsorption rate. The adsorption pump consists of a charcoal coated surface adsorbing ³He-molecules when it is cooled below 20 Kelvin. Its temperature can be regulated with an additional heater and by adjusting the flux rate of liquid ⁴He flowing through a tube nextby. A picture of the ³He-system used for the measurements presented in this theses is given in Fig. 4.1(b).

In low-temperature measurements one has to be careful, because the physical temperature of the device, i.e. the electron temperature, can be strongly elevated above the bath temperature of the cryostat due to heating by high frequency (microwave) electromagnetic radiation. Furthermore, hot photons can activate electron traps in semiconductor devices leading to enhanced 1/f noise [70]. In our ³He-System, an effective microwave cryofiltering is achieved with the help of thin 'lossy' coaxial cables and

an RF-shielded experimental box at low temperatures [Fig. 4.1(a)]. The high frequency attenuation of the coaxcables is due to the Skin effect: at high frequencies the dissipative part of the impedance of the coax drastically increases with frequency. The 40 cm long Thermocoax[©] from *Philips* used in the ³He-cryostat yields an attenuation of 20 dB at $\nu = 1$ GHz $(h\nu/k_B \sim 48 \text{ mK})$ [71]. At high enough frequencies the coax starts to act as a waveguide when the wave length becomes comparable to the crosssection. In this case the attenuation saturates and is estimated to be 52 dB at 1 THz $(h\nu/k_B \sim 48 \text{ K})$.

At the top of the cryostat all wires are filtered at room temperature with additional commercial π -filters. The measured frequency response of these filters is given in Fig. 4.3. The attenuation is ≥ 40 dB up to 3 GHz.



Figure 4.1: (a) Low temperature filtering of in insert of the ³He-cryostat. Here, the tube of the isolation vacuum has been removed. (b) ³He-cryostat with measurement rack.

4.2 Low-frequency noise detection



Figure 4.2: (a) Conventional noise measurement circuit with only one amplifier. (b) Correlation technique with two independent amplifiers (a,b) in parallel. This method offers the advantage of an increased sensitivity. The uncorrelated fluctuations of amplifier A and B are after multiplying as often positive as negative, while the correlated contribution in both channels, stemming from the sample, are after multiplying always positive and dominate over the uncorrelated contributions after long integration times.

4.2 Low-frequency noise detection

Conventionally, the voltage noise fluctuations $\langle \Delta V_S^2 \rangle$ across a sample of resistance R and temperature θ are detected by sending the output of a low-noise amplifier to a fast Fourier transform (FTT) spectrum analyzer, which gives the spectral density of the total voltage noise in a bandwith Δf [Fig. 4.2(a)]. The square of the total voltage noise is

$$\langle \Delta V^2 \rangle = \langle \Delta V_S^2 \rangle + (R + R_L)^2 \Delta I_A^2 + \Delta V_A^2 + 4k_B \theta_L R_L \Delta f \tag{4.1}$$

with R_L the resistance of the leads at temperature θ_L , and ΔI_A^2 and $\Delta V_A^2 = \Delta V_{A,0}^2(1 + f_1/f)$ the current and voltage noise of the amplifier. Below a characteristic frequency f_1 the amplifiers show 1/f voltage noise. Thus, the detection of the voltage noise across the sample requires the knowledge of the noise characteristics of the amplifier ($\Delta I_A^2, \Delta V_{A,0}^2, f_1$) and of the resistance



Figure 4.3: Measured frequency response of a commercial RF-filter (π -filter) at 300 K (50 Ω terminated) used in the measurement setups of the ³He- and ⁴He-kryostat.

4 Measurement techniques

and temperature of the leads. In order to get rid of these uncertainties we used here a cross-correlation technique [see Fig. 4.2(b)]. Measuring the voltage noise with two amplifiers in parallel and multiplying the outputs, the voltage noise contribution of the leads and of the amplifiers is eliminated because these contributions are completely uncorrelated in time. For the cross spectrum of the two output voltages of amplifier a and b we obtain

$$\langle \Delta V_a \Delta V_b \rangle = \langle \Delta V_S^2 \rangle + R^2 \Delta I_A^2 \tag{4.2}$$

for $R_L \ll R$, which is the case in our setup $(R_L \simeq 25 \ \Omega)$. Thus, the determination of the voltage noise $\langle V_S^2 \rangle$ originating from the sample requires to know the current noise of the amplifiers.

The experimental setup for measuring electrical noise in the ³He-system is shown in Fig. 4.4. The measured device is current biased using a floating DC-voltage source together with two high ohmic series resistors $R_S \gg$ R_{sample} . In order to minimize their thermal noise they are mounted directly on top of the sample holder at 270 mK. The leads used to detect the voltage noise fluctuations are only filtered at low temperatures, because the capacitance of the π -filters of 10 nF would affect the bandwith drastically. At the top of the cryostat they directly enter a nearby RF-shielded box [see also Fig. 4.1(b)] where the two low-noise voltage amplifiers (EG&G 5184) are mounted inside. The total capacitance C of the noise leads is $\sim 550 \text{ pF}$ which gives a cut-off frequency $\nu_0 = (2\pi RC)^{-1}$ of ~ 30 kHz for a 10 k Ω sample resistance. The amplifiers, which have a gain of 1000, are driven by two independent sets of batteries to prevent any cross-talk between them. The outputs are filtered with additional π -filters. Finally, the two signals are fed into a two-channel spectrum analyzer (HP 89410A) which calculates the cross-spectrum taking the fast Fourier transform of the two signals separately and multiplying the results together. The achieved sensitivity for voltage noise measurements is of the order 5×10^{-21} V²s.

4.2.1 Calibration

Due to the low-temperature filtering the measured voltage noise can be attenuated which depends on the measurement frequencies and the sample

thermocoax-cable	230 pF
wireing from 1 K to 300 K	$50 \ \mathrm{pF}$
low noise-cable	110 pF
coax-cable inside RF-box	$90 \ \mathrm{pF}$
EG&G preamplifier	$2{\times}35~\mathrm{pF}$

Table 4.1: The independently measured capacitance $\sim 550 \text{ pF}$ is in good agreement with the capacitance determined from a low-pass fit of the measured signal suppression factors.





Figure 4.4: Noise measurement setup. The two voltage amplifiers (EG&G 5184), driven by two independent blocks of batteries, are built into an RF-shielded box. All leads are filtered at low temperatures by lossy thermocoax cables. In addition, π -filters are used at room temperature for the 'normal' leads, whereas the noise leads are RF-shielded up to the amplifiers housed in the RF-tight box. All outgoing leads from this box are again filtered with π -filters. The differential resistance of the point contacts is detected using Lock-In technique with a small AC-current coupled into the electronic DC-circuit by a passive 1:4-tranformator (fully μ -metal shielded). The sample can be illuminated at low temperatures by an infrared diode mounted on the sample holder, which helps to increase the density of the 2 DEG.

resistances. In order to obtain an absolute value for the measured voltage noise S_V the mesurement setup is calibrated by measuring the equilibrium voltage noise at different bath temperatures. Since the sample resistance R and the bath temperature θ are known the Nyquist relation $S_V = 4k_B\theta R$ can be used to obtain the voltage gain as well as the offset noise $S_I^{amp}R^2$ caused by the finite current noise S_I^{amp} of the amplifiers¹. An example for such a temperature calibration is given in Fig. 4.5(a). At low enough frequencies nearly 100 % of the signal is obtained [Fig. 4.5(b)]. Typical measurement frequencies for a 10 k\Omega sample resistance are around 5 – 8 kHz with a typical frequency bandwith of ~ 1 kHz.

¹The current noise due to the amplifiers is of the order 80 fA/ $\sqrt{\text{Hz}}$.

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Figure 4.5: (a) Temperature calibration: the measured equilibrium voltage noise S_V is plotted versus the bath temperature. Using the Nyquist relation $4k_B\theta R$ the signal attenuation due to the filtering as well as the offset noise can be determined very accurately. (b) Voltage noise signal as a function of νR with ν the center of the frequency window, in which the noise is measured, and R the sample resistance. Typical measurement frequencies lie between 5 and 120 kHz and R varies from 1 to 45 k Ω . The solid curve is a fit corresponding to a 1st-order low-pass filter. This fit yiels a capacitance of 535 pF in very good agreement with the independently measured capacitance of 550 pF.

4.2.2 dV/dI-Correction

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The resistance of a QPC in a two dimensional electron gas is electrostacially defined by the voltage which is applied to the split gate as described in section 2.2.2. If the applied bias voltage across the QPC is high enough (out of linear response) the differential resistance can slightly change as shown in Fig. 4.6(a), because lower lying modes, which are not transmitted in equilibrium, start to contribute to the current. As a result the differential resistance dV/dI is not constant but slightly changes with the bias current [72]. The slight asymmetry of the dV/dI [Fig. 4.6(a)] might be due to an asymmetric lithographic definition of the QPC [see Fig. 8.4(b)]. Consequently, the noise data are asymmetric, too, as shown in Fig. 4.6(b), because the offset due to the finite current noise of the amplifiers S_I^{amp} is not constant but varies with the applied bias:

$$S_V^{meas} = S_V + S_I^{amp} \cdot \left(\frac{dV}{dI}\right)^2.$$
(4.3)

If the changes in the dV/dI are not too large, this can be corrected subtracting the contribution which is due to the amplifier noise. That is shown





Figure 4.6: (a) Differential resistance dV/dI as a function of the bias current. dV/dI changes only within 3 % and is slightly asymmetric. (b) Current noise vs bias current. After the subtraction of the amplifier noise as described in this subsection, the data become perfectly symmetric. The asymmetry in the uncorrected data is due to the asymmetric dV/dI. Comparing the absolute values of noise there is a huge offset between the corrected and the raw data.

in Fig. 4.6(b) which represents a drastic example. In principle, one could also try to feed back a signal to the gate voltage, such that the differential resistance remains constant [36], however, for more than one point contact in series this is impossible. Nevertheless, the best thing is to measure noise in a bias regime where dV/dI is constant to make sure that one stays in the regime of linear response. For the noise measurements presented in the following chapters the differential resistance has always been measured in addition.

4.2.3 Analysis of noise data

Typically, a few 100 voltage noise spectra are taken for a given bias current, which are all averaged. Each spectrum consists of 401 points continuously distributed over the chosen frequency interval. The measurement program calculates the average noise power of the averaged spectra excluding points which are off from the average by a certain magnitude, which is manually adjusted. However, there is also another method for obtaining the average noise power which is less sensitive to 'accidental' peaks in the spectra [see Fig. 4.7]. In this method a histogram of the averaged noise spectrum is calculated for each bias point [73]. The center of a Gaussian fit of these





Figure 4.7: (a) Example of a voltage noise power spectrum between 18.3 and 25.8 kHz. (b) Histogramm of the trace above. The histogram is fitted with a Gaussian function in order to determine the average noise.

histograms can give a better estimation for the average noise power than just the average of the spectrum, since peaks, which appear in the spectra, do not very much contribute in the histogram. This alternative method allows one to use a larger frequency window so that the observation time is reduced. For the experiments presented in the following chapters we tried to measure noise in a frequency window without any spikes and used the first method to analyze the noise spectra.



Chapter 5

The Hanbury Brown and Twiss experiment with Fermions

5.1 Introduction

In contrast to classical particles, identical quantum particles, e.g. identical photons or electrons, are inherently indistinguishable. This is because the laws of quantum mechanics do not allow to label identical quantum particles (for example by coloring) and to follow their trajectories at the same time. The identity of quantum mechanical particles manifest itself in higherorder correlation functions as for example in the second-order correlations of the time dependent fluctuations of the electrical current. The first measurements in the field of quantum statistics, which is devoted to the study of identical quantum particles, were carried out with photons by Hanbury Brown and Twiss (HBT) in the 1950s. In a pioneering experiment HBT determined the size of astronomical radio sources by measuring the spatial coherence of the emitted radiation from correlations between *intensity* fluctuations at two different locations [74]. Their setup is shown in Fig. 5.1(a) where two reflectors (R) combined with photo multipliers (Ph) spaced by a distance d were used to detect the deviations $\Delta I(t)$ of the time dependent intensity I(t) from its mean value $\langle I \rangle$. For separations d smaller than the spatial coherence length l_{coh} of the detected radiation a *positive* correlator $\langle \Delta I_1 \Delta I_2 \rangle$ between the intensity fluctuations measured by telescope 1 and 2 was found, whereas in the case of $d > l_{coh}$ the flucutations ΔI_1 and ΔI_2 were completely independent. With this method the angular diameter α of several stellar objects was measured down to $\simeq 0.0005''$ using the relation $\alpha \cdot l_{coh} = \alpha \cdot d \simeq \lambda.$ The intensity interferometry introduced by HBT radically differs from that of the phase sensitive Michelson stellar interferometer [see Fig. 5.1(b)], since measuring the correlation between *intensity*





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Figure 5.1: (a) Schematics of the Hanbury Brown and Twiss stellar intensity interferometer and (b) the phase sensitive Michelson star interferometer, both used to determine the angular diameter of nearby stellar objects (after Ref. [75]).

fluctuations means that the relative phases of the two signals are lost¹. The fact that the information 'hidden' in the relations between the phases of the two signals can nevertheless be obtained from intensity correlations is due to the specific statistics of a thermal field of photons obeying Bose-Einstein statistics, as we will discuss below.

Subsequently, a table-top version of the HBT experiment invoked one optical light source (Hg vapor lamp) and two detectors measuring the fluctuations in the transmitted (T) and reflected beam (R) generated at a half-transperent mirror (beam splitter) [see Fig. 5.2(a)]. Instead of the spatial coherence, in such an experiment the temporal coherence of the light beam is probed. The equal-time intensity correlation between the two separated photon streams was found to be positive, too [77, 78, 79, 80]. At the core of these two observations - positive intensity correlations - lies the fundamental property of the particles making-up the light (= photons), namely that they are *bosons*, which can occupy the same state. In an oversimplified picture bosons do have a tendency to bunch in clusters ('photon bunching') [Fig. 5.2(b)]. Thus, several particles reach the beam splitter at the same time, so that the probability to detect one particle in the transmitted and one in the reflected beam simultaneously is enhanced and a positive correlation will be measured. The situation dramatically changes if the bosons

¹In advantage to the Michelson technique, the base-line in the HBT setup is much less limited by atmospheric or instrumental fluctuations, so that the resolution can be much larger. Nowadays, intensity interferometry has become an important tool in high energy nuclear and particle physics [76].





Figure 5.2: (a) Schematics of a HBT experiment. (b) Particles obeying Bose-Einstein statistics tend to cluster (bunching). Thus, a positive correlation is observed between fluctuations in the transmitted and reflected beam. (c) In contrast, the particles in a degenerate beam of fermions expel each other (antibunching) because a state can be occupied by only one fermion. As a consequence, the fluctuations in the partial beams are expected to be fully anticorrelated.

in the HBT experiment were replaced by fermions which cannot occupy the same state and avoid each other ('antibunch') because of the Pauli principle. Such a beam of degenerated fermions does not show any fluctuations in the number of particles (see below) and in a HBT-type experiment with fermions, a negative correlation between intensity fluctuations in the transmitted and reflected beam is expected [Fig. 5.2(c)].

This chapter describes one successful realization of the electronic counterpart of the Hanbury Brown and Twiss experiment performed in a semiconducting device. Another succesful realization of a different experimental approach, also in a semiconducting environement, was simultaneously and independently performed by Oliver *et al.* [81]. Although there were earlier attempts to measure the expected negative correlations in a free electron beam, the problem to overcome is that even in the best field emission source the particle density is very low $(10^{-6} \text{ per mode})$ due to Coulomb repulsion and the spreading of the beam. This made it impossible to measure the anti-correlation before [82, 83].

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5.2 Statistics and shot noise

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The mean occupation number $\langle n\rangle$ of a single particle state with energy E is given by

$$\langle n \rangle = \left[\exp\left(\frac{E-\mu}{k_B\theta}\right) + a \right]^{-1}$$
 (5.1)

with a = 0 for classical particles, a = +1 for a gas of indistinguishable fermions and a = -1 for a gas of identical bosons. For fermions, $\langle n \rangle$ never exceeds unity because the occupation number n itself can only be 0 or 1 due to the Pauli-principle. In the following we consider the statistical fluctuations of the occupation number n around its mean value $\langle n \rangle$. The mean square fluctuations are²

$$\langle \Delta n^2 \rangle \equiv \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle \Big(1 - a \langle n \rangle \Big).$$
 (5.2)

In the classical case with particles obeying Maxwell-Boltzmann statistics (a = 0), the fluctuations just equal the mean occupation number: $\langle \Delta n^2 \rangle = \langle n \rangle$. For fermions (a = 1) the fluctuations start to vanish as the mean occupation number $\langle n \rangle$ reaches unity. Thus, a fully occupied fermionic beam with $\langle n \rangle = 1$ is noiseless, indicating the complete absence of observable particle nature. In general, statistics for which $\langle \Delta n^2 \rangle < \langle n \rangle$ are called sub-Poissonian. For particles obeying Bose-Einstein statistics (a = -1), the fluctuations can be larger than in the classical case: $\langle \Delta n^2 \rangle > \langle n \rangle$. Such statistics are therefore denoted as super-Poissonian.

In Fig. 5.3(a) the probability p(n) that there are exactly n particles in a state of energy E is plotted as a function of the particle number nfor the three different kinds of statistics³. In case of degenerate fermions this probability is 1 for n = 1 and zero otherwise: the state is occupied only once with probability one. Classically, p(n) equals a Poissonian distribution that achieves a maximum value for $n = \langle n \rangle$. Surprisingly, the probability to have exactly zero particles in a specific state E is in the case of Bose-Einstein statistics larger than for any finite number n. This means that the fluctuations in this case are maximal. The ratio p(n)/p(n-1), which measures the probability to aquire one more particle to the state E already occupied by n-1 particles, is independent of the number of particles already occupying that state. This behaviour is what we call 'bunching'. In contrast, fermions exhibit a negative statistical correlation ('antibunching').

²For details, see: R. K. Pathria, Statistical Mechanics, Pergamon Press, Oxford (1986). ³In two dimensional systems, other fractional statistics are possible, too. The particles obeying fractional statistics are called anyons.





Figure 5.3: (a) Probability p(n) that there are *exactly* n particles in a state of energy E as a function of the particle number n for different statistics. From the relation $\delta E \cdot \tau \sim \hbar$ the distribution p(n) can also be regarded as the probability to detect n particles simultaneously in the two channels of a HBT-type experiment. (b) Overview over different physical systems of bosons and electrons with various statistics.

Photons do not necessarily 'bunch', because the property 'bunching' depends on the particle statistics. It has been shown that it is also possible to generate photon fields with a small mean particle number $\langle n \rangle$ per mode volume displaying 'photon antibunching'! [84, 85, 86, 87] In a recent experiment single photons were triggered periodically using a single quantum dot excited on resonance by laser pulses. The generated trains of photons show pronounced anti-bunching [88].

In a HBT type experiment, *not* the fluctuations $\Delta n(t) = n(t) - \langle n \rangle$ of the particle number n(t) (shot noise) in a *single* beam are detected, but the correlation between fluctuations in the transmitted (T) and reflected (R) beam originating from a beam splitter [Fig. 5.2(a)]. If the incident

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statistics	$\langle \Delta n^2 \rangle$	cross-correlation
Poissonian	$=\langle n \rangle$	= 0
SUPER-Poissonian ('bunching')	$>\langle n \rangle$	> 0
SUB-Poissonian ('antibunching')	$<\langle n \rangle$	< 0

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 Table 5.1: Overview over different types of statistics and the cross-correlations

 between fluctuations in the transmitted and reflected beam in a HBT-type experiment.

beam (IN) is not prepared at a single-particle level, but shows intensity fluctuations $\Delta n(t)$, the correlator $\langle \Delta n_t \Delta n_r \rangle$ between the fluctuations in the transmitted Δn_t and the reflected Δn_r beam is given by⁴:

$$\langle \Delta n_t \Delta n_r \rangle = t(1-t) \cdot \left\{ \langle \Delta n^2 \rangle - \langle n \rangle \right\},$$
 (5.3)

where t is the transmission probability of the beam splitter. The crosscorrelation in Eq. (5.3) is a sum of a term proportional to $\langle \Delta n^2 \rangle$, which depends on the particle statistics in the incident beam, and a term proportional to $\langle n \rangle$, which is caused by the probabilistic partitioning of single particles at the beam splitter. This second term always gives a negative contribution to the cross-correlation independent of whether the particles are bosons or fermions. The auto-correlation of the transmitted (or reflected) beam equals

$$\langle \Delta n_t^2 \rangle = t(1-t) \cdot \left\{ \frac{t}{1-t} \langle \Delta n^2 \rangle + \langle n \rangle \right\}.$$
(5.4)

Here, the sign of the second term is positive in contrast to the crosscorrelation in Eq. (5.3). For an incident beam of particles obeying *Pois*sonian statistics with $\langle \Delta n^2 \rangle = \langle n \rangle$ the cross-correlation (5.3) is zero. For super-Poissonian statistics, $\langle \Delta n^2 \rangle > \langle n \rangle$, the cross-correlation is positive (HBT result for thermal light). In contrast, anticorrelation results if the noise in the incident beam is sub-Poissonian: $\langle \Delta n^2 \rangle < \langle n \rangle$. Maximal anticorrelation is obtained if the incident beam carries no fluctuations at all $\langle \langle \Delta n^2 \rangle = 0 \rangle$, which is the case for a completely degenerated electron beam at zero temperature. Noise suppression in a single beam due to the fermionic nature of the charge carriers has been found in electrical noise measurements on quantum-point contacts and nanowires [36, 43, 47, 48, 49, 62, 89]. Especially, the shot noise suppression measured on ballistic quantum point contacts which was found to be in excellent aggreement with the theoretical expection $S_I = 2e|I|(1 - T)$ already demonstrates the absence of noise in a degenerate fermionic beam [see subsect. 2.4.1].

 $^{{}^{4}}A$ derivation of the expressions Eq. (5.3) and Eq. (5.4) is given in appendix A.





Figure 5.4: Intensity correlation experiment for a degenerated beam of electrons realized in a Hall bar. A metallic split gate serves as tunable beam splitter [see Fig. 5.5(b)]. Electrons escaping from contact 1 travel along the upper edge until reaching the beam splitter, where they are either reflected into contact 3 or transmitted to contact 2. The time-dependent transmitted and reflected currents $I_{t,r}$ are converted to voltage signals by two 1 k Ω series resistors R_S . Finally, the voltage fluctuations across the two resistors are detected by two amplifiers and the two signals are fed to a spectrum analyzer, which calculates the cross-correlation spectrum.

5.3 Realization in a semiconducting environment

Theoretically, HBT-type experiments with fermions and other multi-terminal correlation experiments have been considered by various people before any experiment has been performed [33, 35, 59, 90, 91, 92, 93].

One way to realize a HBT experiment with electrons has been proposed by Büttiker [35]: In a two dimensional electron gas in the quantum Hall regime [sect. 2.2.1] the current flows in one-dimensional channels along the edges of the device [Fig. 5.4]. These edge channels can be used to separate the incident from the reflected beam. If there would be no magnetic field the current would also flow in the bulk and incident and reflected beam





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Figure 5.5: (a) Transmitted and reflected current measured at contact 2 and 3, respecitvely, as a function of the gate voltage. At 2.5 K the two edge states are spin-degenerated. (b) Scanning electron microscope image of the beam splitter. Only one of these three pairs of split gates has been used.

could not be distinguished. The beam splitter consists of a lithographically patterned metallic split gate, which can be tuned by a negative gate voltage.

Applying a constant voltage V to contact 1 the charge current I is injected into the Hall bar. The magnetic field perpendicular to the 2 DEG is adjusted to filling factor $\nu = 2$, so that the current flows in one spindegenerated edge state. The electrons escaping from contact 1 travel along the upper edge until reaching the beam splitter, where they are either transmitted with probability t to leave the device at contact 2, or reflected with probability r = 1 - t leaving at contact 3. Provided $eV \gg k\theta$, the theory predicts for the spectral densities of the auto- and cross-correlation according to Eq. (5.3) and (5.4) [33]

$$\langle \Delta I_{\alpha} \Delta I_{\beta} \rangle_{\omega} = \pm 2e|I|t(1-t) \tag{5.5}$$

with α , β either t or r [see app. A]. The positive sign corresponds to the auto-correlation, where $\alpha = \beta$, and the negative one to the cross-correlation with $\alpha \neq \beta$. Because the cross-/auto-correlation is largest for t = 1/2 the beam splitter is adjusted to transmit and reflect electrons with 50 % probability [Fig. 5.5].





Figure 5.6: Measured spectral densities of correlations between current-fluctuations vs the current I of the incident beam at 2.5 K. The offset noise arising from thermal fluctuations and residual amplifier noise has been subtracted. The absolute slopes are $(0.23 \pm 0.030) \cdot 2e$ and $(0.26 \pm 0.037) \cdot 2e$ for the auto-correlation and cross-correlation, respectively, in agreement with the expected prefactor t(1-t) = 1/4.

5.4 'Full antibunching'

Fig. 5.6 shows the cross-correlation $\langle \Delta I_t \Delta I_r \rangle_{\omega}$ (solid squares) of the fluctuations ΔI_t and ΔI_r versus bias current I at T = 2.5 K. A nearly linear dependence with a negative slope is found showing that the fluctuations are indeed anticorrelated. The auto-correlation (solid circles) of the transmitted current $\langle (\Delta I_t)^2 \rangle_{\omega}$ (or reflected current, not shown) has a positive slope. The negative cross-correlation and the positive auto-correlation are equal in magnitude confirming that the partial beams are fully anticorrelated. We can therefore conclude that there is no uncertainty in the occupation of the incident beam are occupied with probability one and hence are noiseless by virtue of the Pauli principle. Formally, this follows also from $\langle \Delta n^2 \rangle = \langle (\Delta n_t + \Delta n_r)^2 \rangle = \langle \Delta n_t^2 \rangle + 2 \langle \Delta n_t \Delta n_t \rangle + \langle \Delta n_r^2 \rangle = 0$ within experimental accuracy. The fact that the current I of the incident beam is noiseless demonstrates that the constant voltage applied to reservoir 1 is





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Figure 5.7: The current injected at contact 1 can be depleted with an additional gate p. For p close to one the statistics is fermionic while for p close to zero it changes to classical Poisson statistics. Note, that the current incident to the beam splitter equals the current I flowing into contact 1.

converted into a *constant* current $e^2 V/h$ (per accessible mode) according to the fundamental requirement of the Landauer-Büttiker formalism [sect. 2.1].

5.5 Sensitivity to the occupation in the incident beam

In an extension of our experiment we have changed the statistics in the incident beam using an additional gate with transmission $p \in [0, 1]$. If $\langle n \rangle$ is the mean particle number of the beam incident to the beam splitter, the particle number behind the first gate p is given by $\langle \tilde{n} \rangle = p \langle n \rangle$ with noise $\langle \Delta \tilde{n}^2 \rangle = p(1-p) \langle n \rangle$. Using Eq. 5.3 and 5.4 the normalized auto- and cross-correlation depend on the transmission probability p as

$$\frac{\langle \Delta I_t^2 \rangle_{\omega}}{2e|I|} = t(1-pt) = \frac{2-p}{4}$$
(5.6)

$$\frac{\langle \Delta I_t \Delta I_r \rangle_{\omega}}{2e|I|} = -t(1-t)p = -\frac{p}{4}, \qquad (5.7)$$

for t = 1/2 [see also app. A]. If p decreases from 1 to 0 the states in the incident beam are diluted and the anticorrelation (5.7) becomes smaller ['+-centered' and open squares in Fig. 5.8]. In case of very low transmission p the statistics in the incident beam is Poissonian and the anticorrelation disappears as discussed above. The auto-correlation itself increases, because of the increased noise in the incident beam [see Eq. (5.4)]. The dependence of the auto- and cross-correlation on the probability p is shown in Fig. 5.9. The measured slopes of the data in Fig. 5.8 are in good agreement with the predictions of Eq. (5.6) and (5.7) within experimental accuracy.





Figure 5.8: If the incident beam is diluted by lowering the transmission probability p from 1 to 0, the cross-correlation gets smaller and disappears for p close to zero. The auto-correlation itself increases, because the incident beam is noisy.

5.6 'Negative' thermal noise

The two experiments presented in sect. 5.4 and 5.5 probe the system strongly out of equilibrium $(eV \gg k_B\theta)$. Thermal fluctuations are therefore negligible. Interestingly, theory predicts that thermal fluctuations at two different reservoirs can be correlated, too [33]. The correlation of thermal fluctuations between two different reservoirs α and β are

$$\langle \Delta I_{\alpha} \Delta I_{\beta} \rangle_{\omega} = -2k_B \theta \, G_0 \cdot (t_{\alpha\beta} + t_{\beta\alpha}) \tag{5.8}$$

with $t_{\alpha\beta}$ ($t_{\beta\alpha}$) the direct transmission probability from reservoir β to α (α to β). According to this expression the thermal fluctuations at different contacts should also be anticorrelated, provided that $t_{\alpha\beta}$ and $t_{\beta\alpha}$ are not both zero. In contrast to the previous non-equilibrium experiments, this anticorrelation is not specific to the statistics of the charge carriers, but is





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Figure 5.9: Dependence of the cross- and auto-correlation on the transmission probability p of the additional gate for t = 1/2. If p is lowered from 1 to 0 the statistics in the incident beam are changed from Fermi-Dirac to Poisson statistics. The normalized auto-correlation $\langle \Delta I_t^2 \rangle_{\omega}/2eI$ increases from 1/4 to 1/2 and the normalized cross-correlation $\langle \Delta I_t \Delta I_r \rangle_{\omega}/2eI$ from -1/4 to 0 in good agreement with the prediction of Eq. 5.6 and 5.7.

a consequence of charge conservation. In a multi-terminal device different contacts might not be coupled via a finite direct transmission probability, leaving thermal noise at these contacts completely uncorrelated [33].

Fig. 5.10(a) shows the schematics of a device used to demonstrate the prediction of Eq. (5.8) experimentally. In the first case, the switch at contact 4 is open. Thus, any charge transmitted from contact 2 to 4 is reinjected by this contact because contact 4 is floating. For this reason, contact 4 can be disregarded if the switch is open and t_{32} equals the transmission t of the split gate. In the opposite direction direct transmission vanishes $(t_{23} = 0)$ because any fluctuations originating from contact 3 are absorbed by contact 1 which is connected to ground. Thus, the correlation between thermal fluctuations at resevoir 2 and 3 is expected to equal

$$\langle \Delta I_2 \Delta I_3 \rangle_\omega = -2k_B \theta G_0 t. \tag{5.9}$$

The correlation between thermal fluctuations are now proportional to the transmission probability t and to the temperature θ . Fig. 5.10(b) shows the measured correlations for three different temperatures. The fluctuations are indeed negatively correlated and proportional to the transmission probability t. The measured magnitudes also agree with theory within the experimental accuracy. Contact 4 may also be connected to ground. This cancels any direct transmission from contact 2 to 3 ($t_{32} = t_{23} = 0$). Thus, the correlations are destroyed and both reservoirs fluctuate independently.





Figure 5.10: (a) Four terminal conductor in the integer quantum Hall regime. The transmission t is again tunable via the applied negative voltage to the split gate. (b) Correlations between equilibrium fluctuations (eV = 0) vs transmission probability t. If contact 4 is open (solid symbols) the thermal fluctuations are negatively correlated. For contact 4 connected to ground resevoir 2 and 3 fluctuate independently.

This is confirmed by the open circles in Fig. 5.10(b) which are data obtained for contact 4 grounded at a relatively large temperature of 7.5 K.

5.7 Conclusions and Outlook

In conclusion, a HBT type experiment with electrons has been realized in a solid state device. This experiment measures the equal-time correlations between fluctuations in the transmitted and reflected beam, but does not allow to extract any information about the correlation time of the electrons. This would require shot noise experiments at much higher frequencies (~ THz). Furthermore, we have shown that current correlations are sensitive to the particle statistics in the incident beam. Full anticorrelation is observed for electrons obeying Fermi-Dirac statistics, whereas the anticorrelation is gradually suppressed if the incident beam is diluted.





Figure 5.11: Inelastic scattering along the upper edge of the device could be introduced with an additional floating reservoir 4.

Effect of incoherent scattering

The electronic correlations measured in a HBT experiment can be strongly affected by incoherent scattering. This has recently been shown in a theoretical work of Texier and Büttiker [94]. Interestingly, inelastic scattering can even lead to positive correlations between transmitted and reflected beam, if there are more than two edge states carrying the current [Fig. 5.12]. Inelastic scattering along the upper edge of the decive in Fig. 5.7 is indicated by an additional floating reservoir [see Fig. 5.11]. For $\nu = 4$ in the integer quantum Hall regime we consider the case where the first spin-degenerated edge state passes the point contact p with 100 % transmission probability while the second one is only partially transmitted. Thus, the fluctuations δI_1 in the first edge state are zero while there is partition noise in the second one: $\delta I_2 = \delta I$. The floating reservoir has the affect that it equipartitions the current between the two available edge states. The current fluctuations introduced by the point contact p are now present in *both* edge states. If the second point contact t separates the two states so that the first one is totally transmitted and the second one is totally reflected a positive correlation in the current fluctuations between transmitted and reflected channel can be measured.

HBT type experiments in the fractional quantum Hall regime

Shot noise measurements in the fractional quantum Hall regime have provided direct measurements of the fractional charge of quasiparticles [31, 32]. In a recent theoretical work of Safi *et al.* [95] it is suggested to monitor the





Figure 5.12: Predicted normalized cross-correlation as a function of the transmission probability p and t corresponding to the second Landau level at the first and second point contact, respectively [see Fig. 5.11]. For t = 0 the crosscorrelation is positive.

statistics in the FQHE via a HBT-type experiment, where quasiparticles are emitted from one edge (3) and after tunneling through the correlated Hall fluid are collected into two receiving edges (1,2) [see Fig. 5.13]. If the filling factor ν is lowered from 1 towards 1/4, the noise correlation is reduced from full antibunching at $\nu = 1$ to 0 at $\nu = 1/4$. Amazingly, the correlations become *positive* for $\nu < 1/4$ (i.e. at filling factors 1/5, 1/7, 1/9, ...) reminiscing of bosons bunching together. According to the authors of Ref. [95] this positive correlations can be either attributed to the fact that the fractional statistics are bosonic at $\nu \to 0$, or to the eventual presence of composite bosons resulting from attachment of an odd number of flux quanta.



Figure 5.13: SEM picture of two parallel point contacts. Quasiparticles tunneling from edge 3 through the correlated Hall liquid could be collected at the receiving edges 1 and 2.

5 The Hanbury Brown and Twiss experiment with Fermions

Positive HBT correlations in normal metal-superconductor devices

The observation of bunching of electrons might be possible by preparing electronic states with fluctations larger than the classical Poisson value, which have recently been observed in resonant tunneling devices and in superconducting weak links [52, 55, 96, 97]. Especially, positive (bosonic) correlations are theoretically shown to exist in a fermionic system, composed of a superconductor connected to two normal reservoirs via a narrow scattering region [96] [Fig. 5.14].



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Figure 5.14: Bunching of electrons might be observerd in a system composed of a superconducting injector, where electrons are paired in Copper pairs, and normal conducting 'Y'-shape structure. Such a device could also serve as an entangler for quantum computing [98].

Chapter 6

Shot noise by quantum scattering in chaotic cavities

6.1 Introduction

In classical mechanics, the essential difference between chaos and regular behaviour is the exponential sensitivity to the initial conditions of the system for the former. Each trajectory of a classical chaotic system explores the entire phase space on a time scale given by the ergodic time. In the absence of chaos stable periodic orbits exist. The general aim of the field of quantum chaos is to study the dependence of quantum properties on the chaotic dynamics of the underlying *classical* system, i.e. the relation between classical trajectories and quantum mechanics. In a semiclassical description, the transmission probability through a chaotic cavity, as depicted in Fig. 6.1, is given by the product of an amplitude and a phase $exp(i\phi)$, where ϕ corresponds to the classical action $S = \int dt L(t) dt$. This semiclassical approximation is appropriate in the short-wavelength limit and equals the classical description for $\hbar \to 0$ [for a review, see Ref. [99]]. Another theoretical approach is random matrix theory (RMT). It can be used to describe the statistics of energy levels in e.g. quantum chaotic systems [for a review, see Ref. [17]]. RMT can only be applied when the classical motion in the cavity is fully chaotic, while the semiclassical approach describes also the case of integrable (non-chaotic) cavities.

In high quality semiconductor heterostructures, cavities with dimensions smaller than the elastic mean free path l as well as the phase coherence length l_{ϕ} can easily be defined providing ideal tools to study these theoretical ideas of quantum chaos [Fig. 6.2]. Disorder is not explicitly required

 $^{^1\}mathrm{For}$ a one-dimensional problem this approximation is generally known as the WKB (Wentzel-Kramers-Brillouin) method.





Figure 6.1: (a) A classical chaotic trajectory in an open stadium billiard. (b) Gray scale plot of a quantum wave function for the same structure. Signatures of classical chaos also appear in quantum transport [100].

for chaoticity. Such a device corresponds to a mesoscopic electron billard. There has been a lot of experimental work on quantum transport in chaotic and integrable cavities. In these experiments the quantum transmission probability, intimately related with the conductance, is explored as a function of the incident momentum k or the magnetic field B [see for example Ref. [101, 102]]. These measurements are sensitive to chaotic or regular behaviour. On the other hand, shot noise measurements are able to demonstrate how much 'quantum-like' the chaotic system is. This is due to the fact, that shot noise in mesoscopic systems is a purely quantum mechanical effect [103]. It is the quantum nature of the chaotic electron motion generating noise, classical chaos alone is not sufficient. This is discussed in detail in chapter 7.

The first part of this chapter briefly summarizes the theory on transport and noise in quantum chaotic cavities. In the second part experimental results for shot noise of a fully quantum chaotic system are discussed.

6.2 Transport properties of chaotic cavities

Figure 6.2 shows a chaotic cavity with two openings of the same width connected to reservoirs on the left and the right side. Classically, the averaged transmission probability T for an electron originating from the left and mov-



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Figure 6.2: Cavity electrostatically defined in a two-dimensional electron gas using five metallic top gates. Four of them are used to define the two point contacts, which couple the cavity to the reservoirs to the left and the right side. The fifth large gate is used to alter the shape.

6.2 Transport properties of chaotic cavities



Figure 6.3: (a) Grayscale map of conductance fluctuations $\Delta G(V_G, B)$ as a function of magnetic field and the gate voltage V_G determining the shape of the cavity measured at 270 mK. (b) Shape averaged magnetoconductance. The minimum at B = 0 is due to weak localization. A fit according Eq. (6.2) yields a phase coherence time $\tau_{\phi} \sim 0.2$ ns.

ing to the right lead equals the reflection probability R; i.e. R = T. This changes if the size L of the cavity becomes smaller than the phase coherence length l_{ϕ} . Similar to the case of diffusive systems, the probability for an electron to be reflected can be larger than the transmission probability due to constructive interference of pairs of time-reversed paths ('weak localization'). For an *open* cavity, which is a cavity with $N_{L,R}$ open channels in the left and right contact (i.e. $T_{1...N_{L,R}} = 1$, $T_{>N_{L,R}} = 0$), RMT predicts for the ensemble averaged total conductance G:

$$\left\langle \frac{G}{G_0} \right\rangle_{ens.} = \frac{N_L N_R}{N_L + N_R - 1 + 2/\beta}.$$
(6.1)

The parameter β in Eq. (6.1) refers to the symmetry of the ensemble of scattering matrices. For $\beta = 1$ time-reversal symmetry is present and the scattering matrix of the chaotic cavity is assumed to be a member of the circular orthogonal ensemble (COE), whereas for $\beta = 2$ time-reversal symmetry is broken and the scattering matrix belongs to the circular unitary





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ensemble (CUE). For this latter case the total conductance is equal to the series conductance of the two contacts. In the limit of large $N_{L(R)}$ the weak-localization correction to the conductance vanishes and the total conductance G equals the classical series conductance. The magnetoconductance dip (weak localization) of a fully chaotic cavity centered around zero magnetic field has a Lorentzian line shape [for a review, see Ref. [104]]

$$\langle G(B) \rangle_{ens.} = \langle G(0) \rangle_{ens.} - \frac{\Delta G}{1 + (2B/\alpha\phi_0)^2} \tag{6.2}$$

with $\phi_0 \equiv e/h$ the with $\phi_0 \equiv e/h$ the elementary flux quantum, α^{-1} the area enclosed by a typical trajectory and

$$\Delta G \simeq \frac{e^2}{h} \left(\frac{N}{2N + \gamma_{\phi}} \right). \tag{6.3}$$

 $\gamma_{\phi} = h/\tau_{\phi}\Delta$ with τ_{ϕ} the phase coherence time and $\Delta = 2\pi\hbar^2/m^*A$ the mean level spacing for a dot with area A. $N = N_L = N_R$ denotes the number of modes in the contacts of the cavity. Experimental results illustrating the weak localization effect of the cavity in Fig. 6.2 are presented in Fig. 6.3. In contrast to the Lorentzian line shape (6.2) of a chaotic cavity, integrable cavities show a linear weak-localization dip [104, 105]. In this way chaotic and non-chaotic cavities can experimentally be distinguished [101, 102].

In addition to the weak localization contribution at zero field, interference of coherent electrons within the cavity cause random but repeatable conductance fluctuations ΔG when the magnetic field or the shape are varied [see Fig. 6.3(a) and Fig. 6.4]. A magnetic field has the effect that it changes the phase along each particular path in the cavity, so that the interference and hence the conductance varies when the magnetic field is changed. For chaotic cavities the power spectrum of these conductance fluctuations $\Delta G(B)$ decays via a single exponential, whereas it shows a power law tail for large frequencies with peaks corresponding to stable periodic orbits for non-chaotic ones [101, 102]. 6.3 Shot noise of chaotic cavities



Figure 6.5: (a) In a metallic diffusive wire noise and resistance have the same origin which is random scattering on impurities. (b) In a open chaotic cavity the resistance is only due to the contacts which are noiseless. Shot noise occurs from quantum scattering inside the cavity.

6.3 Shot noise of chaotic cavities

For a quantum wire with an intermediate barrier with energy independent transmission probability T, T = 1/2 for example, the Fano factor F defined as $F = S/S_{Poisson} = S/2e|I|$ equals 1 - T = 1/2 [see sect. 2.4.1]. Although an open and symmetric ($N_L = N_R = N$) chaotic cavity has a mean transmission probability $\langle T \rangle$ of 1/2, too, the Fano factor is predicted to be only 1/4. This result was first derived by RMT which for $N_{L,R} \gg 1$ results in a bimodal distribution function of transmission eigenvalues [see Eq. (2.37)] with $\langle T \rangle = 1/2$ and $\langle T(1 - T) \rangle = 1/8$. Thus, the Fano factor $F = \langle T(1 - T) \rangle / \langle T \rangle$ equals 1/4.

Similar to metallic diffusive wires, where F = 1/3 [39, 48, 49], the Fano factor 1/4 for a chaotic cavity is universal in the sense that it is insensitive to microscopic properties [40, 59, 60, 106]². Nevertheless, there is an important difference between these two systems concerning the origin of resistance and noise. In a diffusive conductor resistance and shot noise are both generated locally at scattering centers, which are homogeneously distributed along the wire [Fig. 6.5(a)]. In an open chaotic cavity resistance and shot noise do have a different origin [Fig. 6.5(b)]. The resistance is due to the fundamental quantum resistance of the contacts. Although the source of resistance, the

 $^{^{2}}$ By 'construction' all RMT results are universal. However, there are geometric corrections, which are sensitive to the position of the leads and thus are sample specific [60]. The non-universal corrections are proportional to the angular opening of the contacts, which is negligibly small for our cavities.





Figure 6.6: Distribution function f_C of a closed (a) and open chaotic (b) cavity. In the case of a closed cavity the inverse tunneling rate is larger than all other time scales so that the cavity can be regarded as a reservoir in between two tunnel junctions.

open contacts do not contribute to noise because electrons are transmitted with unit probability. Shot noise arises *inside* the cavity due to quantum mechanical diffraction which splits the electron wave packet into two or more partial waves leaving the two exits. In the semiclassical approach cavity noise is determined by the average fluctuations of the state occupancy inside the cavity given, at T = 0, by [60]

$$S = 2G \int dE f_C (1 - f_C).$$
 (6.4)

Here, $f_C(E)$ denotes the distribution function inside the cavity, which is homogeneous and isotropic. Such a semiclassical description is appropriate if the inverse dwell time is much larger than the level spacing of the cavity. Because it is phase insensitive, interference effects (e.g. weak localization) are excluded from this theory. The total conductance G is equal to the series conductance of the left and right contact $G_0(N_LN_R)/(N_L+N_R)$ with N_L (N_R) open channels (i.e. $T_{1...N_{L,R}} = 1$, $T_{>N_{L,R}} = 0$). For noninteracting electrons the distribution function in the cavity f_C just equals the weighted average of the distribution functions f_L and f_R in the left and right reservoirs [see Fig. 6.6(b)]. In the symmetric case $N_L = N_R$, i.e. $f_C = \frac{1}{2}(f_L + f_R)$, Eq. (6.4) then yields a Fano factor of 1/4. For very asymmetric contacts $(N_L \gg N_R)$ shot noise approaches zero, since the system can then be regarded as a single contact with N_R open and therefore noiseless channels. The general Fano factor $F \equiv S/2e|I|$ for cavity noise is

$$F(\eta) = \frac{N_L N_R}{(N_L + N_R)^2} = \frac{\eta}{(1+\eta)^2},$$
(6.5)

6.3 Shot noise of chaotic cavities

where we introduce the parameter $\eta \equiv N_L/N_R$ which measures the symmetry of the cavity³.

A chaotic cavity is called *closed* if it is separated from the leads by tunnel barriers. In that case, the dynamics inside the cavity does not play any role and the Poissonian voltage noise of the two contacts adds up resulting in $S_I = 1/2 \cdot 2e|I|$ as for a double barrier junction with Fano factor 1/2 [see also chapter 8].

6.3.1 Finite temperatures and inelastic scattering

The Fano factor Eq. (6.5) is valid for zero temperature and non-interacting electrons. In order to model the experimental reality a theory for the crossover from thermal to shot noise is required which includes interaction effects, too. In this section a theoretical derivation for the noise of a chaotic cavity at finite temperature is given based on a semiclassical description closely following Ref. [60]. The final results which we use for the analysis of the experimental data are Eq. (6.24) valid for non-interacting electrons and Eq. (6.27) for inelastic electron-electron scattering. These two equations describe the crossover from thermal to shot noise. Impatient readers can skip this section.

We consider the most general case of a cavity coupled by several contacts to the reservoirs [see Fig. 6.7]. From conservation of charge, the sum of all in and out flowing currents must be zero:

$$\sum_{m} I_{m} = \sum_{m} G_{m} (V_{C} - V_{m}) = V_{C} \tilde{G} - \sum_{m} G_{m} V_{m} = 0$$
(6.6)

$$F = \frac{N_L N_R}{(N_L + N_R)^2} - \left(1 - \frac{2}{\beta}\right) \frac{(N_L - N_R)^2 + N_L N_R}{(N_L + N_R)^3} - \left(1 - \frac{2}{\beta}\right)^2 \frac{(N_L - N_R)^2}{(N_L + N_R)^4}.$$



Figure 6.7: Chaotic cavity, which is coupled by several contacts of conductance G_m to the reservoirs of potential V_l . The resistance of the cavity is negligible compared to the resistance of the contacts. V_C and θ_C denote the potential and the effective temperature of the electrons inside the cavity and are assumed to be uniform over the whole cavity.

 $^{^3 {\}rm The}$ weak-localization correction for shot noise of an open chaotic cavity can be obtained from RMT in the limit $N_{L,R}\gg 1$ [40]:

6 Shot noise by quantum scattering in chaotic cavities

with $\tilde{G} = \sum_{m} G_{m}$. Thus, the potential V_{C} inside the cavity is given by

$$V_C = \sum_m \alpha_m V_m \quad \text{with} \quad \alpha_m \equiv G_m / \tilde{G}. \tag{6.7}$$

The current I_m through the *m*-th contact can be written as

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$$I_m = (V_C - V_m)G_m = \sum_n (\alpha_n V_n G_m - \delta_{nm} V_m G_m)$$
$$= \sum_n (\alpha_m V_n G_n - \delta_{nm} V_n G_n)$$
$$= \sum_n G_{nm} V_n$$
(6.8)

with the multiterminal conductance G_{nm} given by

$$G_{nm} \equiv (\alpha_m - \delta_{nm})G_n. \tag{6.9}$$

In the zero frequency limit, we can apply the charge conservation argument to the time dependent fluctuations in the current δI_m , too, implying that $\sum_m \delta I_m = 0$. The current fluctuations consist of a source term δI_m^S describing current fluctuations in the contacts and of a contribution arising from fluctuations δV_C of the potential inside the cavity⁴:

$$\delta I_m = \delta I_m^S + G_m \,\delta V_C \tag{6.10}$$

$$\sum_m \delta I_m = \sum_m \delta I_m^S + \tilde{G} \,\delta V_C = 0 \quad \Rightarrow \quad \delta V_C = -\frac{1}{\tilde{G}} \sum_m \delta I_m^S$$

$$\delta I_m = \delta I_m^S - \alpha_m \sum_n \delta I_n^S. \tag{6.11}$$

Since $\langle \delta I_n^S \delta I_m^S \rangle = S_n \cdot \delta_{nm}$ we obtain

$$S_{nm} = \langle \delta I_n \delta I_m \rangle = \left\langle \left(\delta I_n^S - \alpha_n \sum_{n'} \delta I_{n'}^S \right) \left(\delta I_m^S - \alpha_m \sum_{m'} \delta I_{m'}^S \right) \right\rangle$$
$$= \left\langle \left(\sum_{n'} \delta I_{n'}^S \delta_{n'n} - \alpha_n \sum_{n'} \delta I_{n'}^S \right) \left(\sum_{n'} \delta I_{n'}^S \delta_{n'm} - \alpha_m \sum_{n'} \delta I_{n'}^S \right) \right\rangle$$
$$= \sum_{n'} (\delta_{n'n} - \alpha_n) (\delta_{n'm} - \alpha_m) S_{n'}. \tag{6.12}$$

⁴After a time of the order of the dwell time τ_D an electron inside the cavity becomes uniformly distributed and leaves the cavity through contact n with the probability α_n . For times $t \gg \tau_D$ this can be described by an instantaneous fluctuation of the potential δV_C inside the cavity [see Ref. [60]]. The requirement of the conservation of the number of electrons leads to minimal correlations [60] between $G_m \delta V_C$ and δI_m^S . The current is conserved at every instant of time, $\sum_n \delta I_n = 0$, what eliminates the fluctuations δV_C .
6.3 Shot noise of chaotic cavities

The next step is to calculate S_n which is the noise in the contacts. The noise power of the current fluctuations in a two-terminal conductor is given by Eq. (2.32) [34, 35]:

$$S_n = 2G_0 \sum_{k=1}^N \int_0^\infty dE \left[T_{kn} f_n (1 - f_n) + T_{kn} f_C (1 - f_C) + T_{kn} (1 - T_{kn}) (f_C - f_n)^2 \right].$$
(6.13)

with T_{kn} the transmission probability of the mode k from lead n to the cavity.

Non-interacting electrons: The distribution function $f_C(E)$ inside the cavity in case of cold electrons can be calculated from the conservation of the number of electrons for each energy interval E to E + dE [see Ref. [60]]. The flux of electrons in this energy interval is $J_n(E) = e^{-1}G_n[f_C(E) - f_n(E)]$, so that the current is given by $I_n = \int dE J_n(E)$. Because of charge conservation $(\sum_n J_n = 0)$ it follows immediately that

$$f_C(E) = \sum_n G_n f_n(E) / \sum_n G_n = \sum_n \alpha_n f_n(E) \quad (6.14)$$

with
$$f_n(E) = \left[1 + \exp\left(\frac{E - eV_n}{k_B\theta}\right)\right]^{-1}$$
. (6.15)

Interacting electrons: If there is strong inelastic scattering inside the cavity, the electrons equilibrate due to energy exchange. The distribution function f_C inside the cavity is then a Fermi-Dirac distribution at an elevated electron temperature $\tilde{\theta}$, which can be calculated from the energy balance equation using the Wiedemann-Franz law. The total heat flow from the reservoirs to the cavity is [107]

$$Q_C = \frac{(k_B \pi)^2}{6e^2} \sum_n G_n(\theta_n^2 - \tilde{\theta}^2) + \frac{1}{2} \sum_n G_n(V_C - V_n)^2.$$
(6.16)

Because of energy conservation, $Q_C = 0$. From this the electron temperature $\tilde{\theta}$ inside the cavity is obtained as⁵:

$$\tilde{\theta}^2 = \theta^2 + \frac{3e^2}{(k_B\pi)^2} \sum_n G_n (V_C - V_n)^2 / \sum_m G_m$$
(6.17)

⁵We assume here, that the bath temperature θ is the same for all reservoirs, i.e. $\theta_n = \theta, \forall n$.

6 Shot noise by quantum scattering in chaotic cavities

$$= \theta^{2} + \frac{3e^{2}}{(k_{B}\pi)^{2}} \sum_{n,m} \alpha_{n} \alpha_{m} (V_{n} - V_{m})^{2}.$$
 (6.18)

The distribution function f_C is then given by

$$f_C(E, V_C, \tilde{\theta}) = \left[1 + \exp\left(\frac{E - eV_C}{k_B \tilde{\theta}}\right)\right]^{-1}.$$
(6.19)

Open cavity:

For an open cavity $T_{kn}(1 - T_{kn}) = 0$ for all modes in the contacts. Each mode is either totally transmitted $(T_{kn} = 1)$ or totally reflected $(T_{kn} = 0)$. In this case, Eq. (6.13) simplifies to

$$S_n = 2 G_0 \sum_k T_{kn} \left[\underbrace{\int dE \ f_n(1 - f_n)}_{= k_B \theta} + \underbrace{\int dE \ f_C(1 - f_C)}_{= k_B \theta_C} \right], \quad (6.20)$$

where the effective temperature θ_C inside the cavity has been introduced. From Eq. (6.12) together with Eq. (6.9) the total noise follows as

$$S_{nm} = 2k_B(\theta + \theta_C) \sum_{n'} G_{n'}(\delta_{n'n} - \alpha_n)(\delta_{n'm} - \alpha_m)$$

= $-2G_{nm}k_B(\theta + \theta_C).$ (6.21)

Non-interacting electrons: For non-interacting (cold) electrons the effective temperature $k_B \theta_C = \int dE f_C (1 - f_C)$ can be calculated by substituting Eq. (6.14) into Eq. (6.21):

$$k_B \theta_C = \int dE \sum_n \alpha_n f_n [1 - \sum_m \alpha_m f_m]$$

=
$$\sum_{n,m} \alpha_n \alpha_m \int dE f_n (1 - f_m)$$

=
$$\frac{e}{2} \sum_{n,m} \alpha_n \alpha_m (V_n - V_m) \coth\left(\frac{e(V_n - V_m)}{2k_B \theta}\right). \quad (6.22)$$

In the second step the relation $\sum_{n} \alpha_n = 1$ has been used. In the particular case that there are only two terminals to the cavity the effective temperature

6.3 Shot noise of chaotic cavities

 $k_B \theta_C$ equals

$$k_B \theta_C = \frac{e}{2} \left[(\alpha_1^2 + \alpha_2^2) \lim_{V \to 0} V \coth\left(\frac{eV}{2k_B\theta}\right) + 2\alpha_1 \alpha_2 V\left(\frac{eV}{2k_B\theta}\right) \right]$$
$$= \frac{G_1^2 + G_2^2}{(G_1 + G_2)^2} k_B \theta + \frac{G_1 G_2}{(G_1 + G_2)^2} eV \coth\left(\frac{eV}{2k_B\theta}\right)$$
(6.23)

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with $V = V_1 - V_2$. From (6.21) the total noise follows as [60]:

$$S = -S_{12} = 2G_{12} \left(k_B \theta + k_B \theta_C \right) = 4k_B \theta G \left\{ 1 + \frac{G_1 G_2}{(G_1 + G_2)^2} \left[\frac{eV}{2k_B \theta} \coth\left(\frac{eV}{2k_B \theta}\right) - 1 \right] \right\}$$
(6.24)

with $G = G_{12} = G_1 G_2 / (G_1 + G_2)$ the total conductance of the cavity. The Fano factor equals [Eq. (6.5)]

$$F \equiv \frac{S}{2e|I|} = \frac{G_1 G_2}{(G_1 + G_2)^2} \qquad (= 0.25 \text{ for } G_1 = G_2).$$
(6.25)

Interacting electrons: In the case of interacting electrons the cavity temperature θ_C for a two terminal chaotic cavity is

$$\theta_C = \tilde{\theta} = \theta \sqrt{1 + \frac{3}{\pi^2 k_B^2} \frac{G_1 G_2}{(G_1 + G_2)^2} \left(\frac{eV}{k_B \theta}\right)^2}.$$
 (6.26)

The noise then $equals^6$

$$S = 2k_B\theta G \left\{ 1 + \sqrt{1 + \frac{3}{\pi^2} \frac{G_1 G_2}{(G_1 + G_2)^2} \cdot \left(\frac{eV}{k_B\theta}\right)^2} \right\}$$
(6.27)

with the Fano factor given by

$$F \equiv \frac{S}{2e|I|} = \frac{\sqrt{3}}{\pi} \frac{\sqrt{G_1 G_2}}{G_1 + G_2} \qquad (\simeq 0.276 \text{ for } G_1 = G_2). \tag{6.28}$$

The functional behaviour of Eq. (6.24) and (6.27) is illustrated in Fig. 6.11.

Closed cavity:

The case of partial reflection of modes at the contacts has to be treated numerically and is discussed in chapter 8.

 $^{^6\}mathrm{To}$ our knowledge, this formula cannot be found in the literature.

6 Shot noise by quantum scattering in chaotic cavities

6.4 The device

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Experimentally, chaotic cavities are realized by two quantum point contacts in series. These are electrostatically defined in a two-dimensional electron gas by metallic split gates on top [Fig. 6.8(a,b)]. The opening of the contacts can be individually tuned by varying the applied gate voltages independently. The 2 DEG forms 80 nm below the surface at the interface of a standard GaAs/Al_{0.3}Ga_{0.7}As-heterojunction. Magnetoresistance measurements yield a carrier density of $2.7 \times 10^{15} \text{ m}^{-2}$, corresponding to a Fermi energy of ~106 K and a mobility of 83 Vs/m⁻² resulting in a mean free path of $\sim 7 \ \mu m$ comparable to the size of the cavity. Three QPCs in series as shown in Fig. 6.8(a) enable to define two cavities of different size: either the outer gates A and C with the middle gate B kept completely open can be used to define a relatively large cavity of $\sim 11 \times 8 \ \mu m$, or 2 of the inner gates (A,B or B,C) creating a smaller cavity of $\sim 5 \times 8 \ \mu m$. The conductance of the QPCs is quantized according to the Landauer formula $G = G_0 \sum_n T_n$ [see Fig. 2.2(a)]. An open cavity is defined when both QPCs are adjusted to a conductance plateau, where N modes are fully transmitted (T = 1)and the others are totally reflected (T=0). The two-terminal conductance G is experimentally found to correspond to the series conductance of the two contacts $G_L G_R / (G_L + G_R)$ with an accuracy of less than 1% [40, 60]. Therefore, direct transmission of electrons from the left to the right contact can be excluded, as well as quantum corrections [109]⁷. Chaoticity in these cavities is due to diffusive boundary scattering and few residual impurities within the cavity. Diffusive reflections will always lead to chaotic dynamics whatever the geometry is.

6.5 Measurements and discussion

Two independent low-noise amplifiers (EG&G 5184) operating at room temperature are used to detect the voltage fluctuations across the cavity. A spectrum analyzer (HP 89410A) calculates the cross-correlation spectrum of the two amplified signals. This technique allows to reduce uncorrelated noise contributions which do not originate from the s ample itself. Furhermore, the whole setup is filtered against RF-interference at low temperatures by a shielded sample-box and lossy microcoaxes to minimize heating by radiation. A detailed description of the experimental setup is given in chap. 4.

Voltage noise is typically measured at frequencies around 6 kHz where the noise is frequency independent (white) up to the maximum bias current

⁷Weak localization can be neglected due to residual magnetic flux through the cavity what is larger than ϕ_0 .





Figure 6.8: (a,b) Scanning electron microscope images of an etched Hall-bar in a two-dimensional electron gas with metallic split gates on top. The fabrication of this device required three optical and three e-beam lithography steps. (c) Electron flow behind a QPC taken from Ref. [108]. The QPC acts as a point source emitting spherical electron waves. Thus, the direct transmission probability from one contact to the other is very low, so that the resistances of the two contacts just add. (d) An open cavity is defined when both contacts are adjusted to a plateau in the conductance curve (1). At position (2) the second mode is partially transmitted, too.





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Figure 6.9: Shot noise of a chaotic cavity with ideal contacts $(G_{L,R}/G_0 =$ integer) for different conductance ratios $\eta = G_L/G_R$. The data for $\eta = 1.0$, 2.7, and 6.2 are offset for clarity by 20, 15 and 5×10^{-28} A²s, respectively. The curves are the theorital predictions according Eq. (6.24) for the measured value of η .

 ≤ 50 nA used in the experiment. Although shot noise is a non-equilibrium phenomenon observed in its purest form in the limit $eV \gg k_B\theta$, in this experiment bias voltages are limited to $\sim 8 \ k_B\theta/e$, only. This is to avoid non-linearities of the current-voltage characteristics of the QPCs [72] and 1/f-noise-contributions occuring at larger currents [36]. Within this limit, the differential resistance, recorded for all noise measurements, changes by less than 2.5 %. The current noise is finally obtained from the measured voltage fluctuations by $S_I = S_V/(dV/dI)^2 - S_I^{amp}$.

Fig. 6.9 shows shot noise measurements of a cavity defined by gates A and B with a size of $\sim 5 \times 8 \,\mu\text{m}$ for different symmetry parameters $\eta = G_L/G_R$. The solid curves describe the crossover from thermal to shot noise for the measured value of η given by Eq. (6.24). In the symmetric case $(\eta = 1)$ with $N_L = N_R = 5$ we obtain a very good agreement between the experimental data and the theoretical prediction of $1/4 \cdot 2e|I|$. When the right contact is further opened $(G_R > G_L) \eta$ increases from 1 (symmetric) to ~ 41 (asymmetric). Thereby, shot noise gradually disappears as expected from Eq. (6.5). The very asymmetric case of large η corresponds to the situation where one contact is widely open, while the other is adjusted to a





Figure 6.10: Shot noise is larger than $1/4 \cdot 2e|I|$ if there is additional partitioning due to non-ideal contacts ($G_{L,R}/G_0 \neq$ integer). The curves are numerical calculations assuming no mode mixing (dashed) and for slight mode mixing of 10% (solid).

conductance plateau and hence does not produce any noise.

For partial transmission in the contacts shot noise is larger than $1/4 \cdot 2e|I|$ because additional noise is generated at the contacts. This is the case when the contacts are not adjusted to a plateau in the conductance curve as for example at position (2) in Fig. 6.8(d). The partition noise of the contacts adds to the cavity noise so that the total noise exceeds $1/4 \cdot 2e|I|$. This is shown in Fig. 6.10 where the first mode in the contacts is fully transmitted $(T_1 = 1)$ while the second one is partially reflected $(T_2 = 0.16)$. The curves are numerical calculations⁸ for no mode mixing (dotted) and for slight mode mixing of ~ 10 % (solid) with $T_1 = 0.90$ and $T_2 = 0.26$.

Up to now we have assumed that inelastic electron scattering inside the cavity can be neglected. In general, heating caused by electron-electron interaction enhances shot noise [25]. The Fano factor of a diffusive wire, for example, changes from 1/3 for non-interacting (cold) electrons to $\sqrt{3}/4$ for interacting (hot) electrons [41]. Heating also affects the shot noise of a chaotic cavity. The Fano factor is modified to [42]:

$$F(\eta) = \frac{\sqrt{3N_L N_R}}{\pi (N_L + N_R)} = \frac{\sqrt{3\eta}}{\pi (1+\eta)},$$
(6.29)

⁸For details, see chapter 8 and app. D.





Figure 6.11: Comparison of the noise of a chaotic cavity $(1/4 \text{ and } \sqrt{3}/2\pi)$ with a diffusive wire $(1/3 \text{ and } \sqrt{3}/4)$ for cold and hot electrons. The arrows indicate the regime in which noise has been measured on chaotic cavities (left arrow) and on diffusive wires (right arrow).

and the crossover from thermal to shot noise is described by Eq. (6.27). For a symmetric cavity $F(\eta=1) \simeq 0.276$ for hot electrons, which is only slighly larger than $F(\eta=1) = 0.25$ for cold electrons.

Fig. 6.11 compares $S_I(eV/k_B\theta)$ in the hot and cold electron regime for a diffusive wire and a chaotic cavity. As is evident, the differences are very small, in particular in the case of a cavity where even a crossing at $eV/k_B\theta \simeq 15$ occurs. In Fig. 6.13(a) the measured noise for $\eta = 1$ of Fig. 6.9 is replotted and compared to the prediction for cold (solid) and for hot electrons (dashed). Although the data points lie clearly closer to the prediction for cold electrons, this alone is not sufficient to decide which regime is realized in the cavity, because of the finite experimental accuracy. An additional criterion is needed.



Figure 6.12: For the estimation of the dwell time of a chaotic cavity.





Figure 6.13: (a) Shot noise of a symmetric cavity and theoretical predictions for cold (solid) and hot electrons (dashed). (b) τ_D/τ_{ee} vs η for the two different types of cavity shown in the inset.

In order to decide whether the cold or hot electron theory is appropriate for the comparison with the measurements, the electron-electron scattering time τ_{ee} is compared with the dwell time for electrons inside the cavity. We argue that thermalization is present if $\tau_D \gg \tau_{ee}$. The average dwell time is the product of the ballistic flight time across the cavity τ_F with the number of scattering events inside the cavity N_{sc} : $\tau_D = \tau_F \cdot N_{sc}$. τ_F is given by $\langle L \rangle / v_F$ with $\langle L \rangle \simeq (a + b)/2$ [see Fig. 6.12] and the number of scattering events N_{sc} by the ratio between the openings and the total surface of the cavity:

$$N_{sc} = \frac{2(a+b)}{W_L + W_R}.$$
(6.30)

With $W_L + W_R = \frac{\lambda_F}{2}(N_L + N_R)$ and $E_F = m^* v_F^2/2$ the dwell time τ_D follows as

$$\tau_D = \frac{4\pi\hbar}{E_F} \left(\frac{\langle L\rangle}{\lambda_F}\right)^2 \frac{1}{(N_L + N_R)}.$$
(6.31)

The electron-electron scattering rate τ_{ee}^{-1} in a two dimensional electron system is given by [110]

$$\tau_{ee}^{-1} = \frac{E_F}{2\pi\hbar} \left(\frac{k_B\theta_e}{E_F}\right)^2 \left[\ln\left(\frac{E_F}{k_B\theta_e}\right) + \ln\left(\frac{2q}{k_F}\right) + 1\right]$$
(6.32)

with the Thomas-Fermi screening wave vector $q = 2me^2/\epsilon_r\epsilon_0\hbar^2$. Because the system is out of equilibrium the temperature θ_e in Eq. (6.32) has to





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Figure 6.14: Fano factor $F \equiv S/2e|I|$ vs the symmetry parameter η for a small cavity (open circles) with widely opened contacts ($\tau_D < \tau_{ee}$) defined by two of the inner gates and for a large cavity (solid squares) with nearly closed contacts ($\tau_D \gg \tau_{ee}$) defined by the two outer gates. The solid and dashed lines are predictions for cold electrons and hot electrons according Eq. (6.25) and (6.28).

be replaced by the effective electron temperature θ_{eff} given by $\theta_{eff} = (1/k_B) \int dE f_C (1 - f_C)^9$. The ratio τ_D / τ_{ee} is plotted in Fig. 6.13(b) as a function of $\eta = G_L/G_R$ for the two different types of cavities with θ_{eff} fixed to the *largest* applied voltage V in the experiment. The upper curve belongs to the large cavity (~ 11 × 8 µm), where the right contact is nearly closed (G_R fixed to G_0). In this case, $\tau_D \gg \tau_{ee}$. The lower curve corresponds to the smaller cavity (~ 5 × 8 µm) with a 5 times larger opening of the right contact. For this type of cavity we find $\tau_D < \tau_{ee}$.

According to this argument we use Eq. (6.27) valid for hot electrons to fit the noise data obtained for chaotic cavities with $\tau_D/\tau_{ee} > 1$. The Fano factor F is the only fitting parameter. On the other hand, we use Eq. (6.24) valid for cold electrons if $\tau_D/\tau_{ee} < 1$. The Fano factors F = S/2e|I| obtained according to this procedure are plotted as a function of the measured η for the two different cavities described above. For the black squares, which belong to the large cavity with nearly closed contacts (large

⁹Thermalization has been assumed for f_C which could slightly overestimate τ_{ee}^{-1} .

6.6 Conclusions

dwell time), we find good agreement with the theoretical Fano factor for hot electrons given by Eq. (6.29) (dashed). The open circles are results for the small cavity with wider opened contacts (small dwell time) which are consistent with the prediction for non-interacting electrons described by Eq. (6.5). If we use the formula for cold electrons instead of the one for hot-electrons to fit the data obtained for the larger cavity, the black squares move only slightly downwards by $\sim 0.02 - 0.03$. They still lie clearly above the open circles, demonstrating that heating is indeed important for the larger cavity. Good agreement between theory and experiment is found for both regimes with the exception of very asymmetric contacts, i.e. $\eta \gg 1$. Here, we attribute the deviations to slight mode mixing within the QPCs, which is difficult to avoid [36]. Let us assume, as an example for the data point at $\eta = 180$, that two modes instead of one participate in the left contact transmitting respectively with $T_1 = 0.97$ and $T_2 = 0.03$ instead of $T_1 = 1.00$ and $T_2 = 0$. This yields a Fano factor of $\simeq 0.06$ in agreement to what is experimentally observed.

6.6 Conclusions

We have experimentally studied shot noise of open chaotic cavities defined by two QPCs in series. In the regime of non-interacting electrons a Fano factor F = S/2e|I| of 1/4 has been measured as theoretically predicted for symmetric cavities. The origin of this shot noise is partitioning of the electron wave function by quantum-mechanical diffraction inside the cavity. The contacts themselves, which actually define the resistance of the system, do *not* contribute to noise. In addition, we have also investigated heating effects due to inelastic electron-electron scattering by changing the opening of the contacts as well as the size of the cavity. Similar to other mesoscopic systems heating increases shot noise in agreement with theory. Shot noise in chaotic cavities is a purely quantum phenomenon. This is discussed in more detail in the following chapter, where an experiment is presented, with which the crossover from 'quantum chaos' to 'classical chaos' can be explored.

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Chapter 7

Quantum-to-classical crossover in noise

7.1 Introduction

It was in 1918 when Schottky theoretically discovered that in vacuum tubes [Fig. 7.1(a)] two types of time dependent current fluctuations (noise) will remain even if all other possible noise sources had been eliminated carefully [26]: the 'Schroteffekt' and the 'Wärmeeffekt'. The latter is known as Nyquist noise, which is due to thermal agitation of electrons, whereas the 'Schroteffekt' is called shot noise [sect. 2.3.2, 2.4]. In Schottky's vacuum tube [Fig. 7.1(a)] electrons e are emitted by the cathode K randomly and independently. Once an electron is emitted it will reach the anode A with certainty. The emission process of electrons is a Poissonian process leading to shot noise given by Schottky's famous formula: S = 2e|I| [Eq. (2.29)]. Early shot noise measurements on vacuum tubes were carried out by Hartmann in 1921 [27] and Hull and Williams in 1925 [28].

In recent years, electrical current noise of coherent conductors (mesoscopic systems) has been studied extensively [25]. In contrast to a vacuum tube, charge does not propagate through free space, but is part of a degenerate and quantum-coherent Fermi sea. Early shot noise measurements on coherent conductors were carried out on quantum point contacts [Fig. 7.1(b)] [36, 43]. In such systems, the electrons emitted from the reservoirs pass the QPC with a certain transmission probability T. As for a vacuum tube the shot noise of a QPC is proportional to the electrical charge and the mean current [34]. One might therefore ask the question whether there is a fundamental difference between Schottky's 'Schroteffekt' and 'modern' shot noise of coherent conductors?

In a QPC the spectral density S equals 2e|I|(1-T) [Eq. (2.34)] so that shot noise is suppressed by the factor 1-T relative to Schottky's result, even

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Figure 7.1: Origin of noise. (a) In a classical vacuum tube the randomness giving raise to the 'Schroteffect' stems solely from fluctuations in the reservoirs, while the transmission through the vacuum occurs with unit probability [see app. C]. (b) QPC as a coherent conductor: at zero temperature randomness is due to scattering within the conductor, only. Although this scattering can generally be classical or quantum-mechanical only quantum scattering generates noise.

predicting zero noise for perfect transmission, which is clearly different to Schottky's result. But what about $T \ll 1$? In this case Schottky's formula S = 2e|I| is obtained for both the vacuum tube and the coherent QPC, although they are distinct systems concerning the origin of noise.

Generally, noise is due to randomness, which can be classical or quantum in nature. Randomness can be inherent in the *reservoirs* emitting charges (emission noise) or in the transmission process between emitter and collector (transmission noise). The 'Schroteffekt' observed in a vacuum tube is solely due to fluctuations of the state occupancy in the reservoirs leading to random emission of electrons while the transmission from cathode to anode through vacuum occurs with unit probability [app. C]. Loosely speaking, the vacuum is noiseless. In an electrical conductor at zero temperature, it is just the other way around. The emission from an ideal reservoir (a Fermi gas) into a quantum wire is noiseless [33] because there is no randomness in the reservoir occupancy due to the sharp Fermi edge present in a degenerate electron gas. Here, randomness is caused by scattering within the conductor, which can be either classical (deterministic) or quantum-mechanical. Both add to the electrical resistance, but only quantum scattering generates noise. This has been conjectured by Beenakker and van Houten [103]. In this chapter we present an experiment which verifies this fundamental 7.2 Theory

statement by measuring shot noise of open chaotic cavities¹.

7.2 Theory

A schematic drawing of a chaotic cavity connected via two QPCs to a left (L) and a right (R) reservoir is given in Fig. 7.2(a). At the contacts charge transport takes place within an integer number N of energy channels with transmission probability T = 1 [see also chap. 6]. Since electrons pass the contacts with unit probability, they are noiseless. Nevertheless, there is a fundamental resistance $R_{L,R}$ associated with each contact: $R_{L,R}^{-1} = G_{L,R} = 2e^2/h \cdot N_{L,R}$ [2, 4]. Here, backscattering is caused by the *chaotic* motion inside the cavity. An electron which enters the cavity from the left contact scatters chaotically inside the cavity and leaves it after the dwell time τ_D , either at the left or the right contact with a probability of 50% averaged over all possible trajectories (symmetric contacts assumed)².

The total resistance R of the chaotic cavity is simply given by the series of the left and right contact resistances: $R_L + R_R$ (the region inside the cavity has negligible resistance). Note that the simple addition law $R = R_L + R_R$ holds irrespectively of whether chaos within the cavity is classical or quantum. As will be shown below this is markedly different for shot noise, the occurrence of which relies on the presence of *quantum* scattering, i.e. diffraction caused by the wave nature of electrons. Because the degree of diffraction can be tuned in the experiment by changing the effective dwell time τ_D , this statement can experimentally be verified.

The shot noise power of an open cavity, at T = 0, can be expressed in terms of the fluctuating distribution function f_C of the electrons inside the cavity [60]:

$$S = 2/R \cdot \int dE \, \langle f_C(\mathbf{p}, \mathbf{r}) [1 - f_C(\mathbf{p}, \mathbf{r})] \rangle.$$
(7.1)

 $\langle \ldots \rangle$ denotes the average over momentum direction \mathbf{p}/p . There are three important time-scales in the problem: 1) the *ballistic flight time* τ_F , which is the time an electron takes to traverse the cavity once; 2) the *dwell time* τ_D ; and, 3) the *quantum scattering time* τ_Q , which qualitatively equals the mean time during which the classical trajectory is 'lost' by diffraction³.

 $^{^1\}mathrm{The}$ cavities are chaotic due to scattering from defects and boundaries. See also chapter 6.

²The cavities are chosen large enough with single electron level spacings $\delta E \ll eV$ so that a sufficiently large ensemble of eigenstates is probed.

³The 'correlation length' $\Delta \varphi_{\lambda}$ of the fluctuations of the distribution function f_C [see Fig. 7.2(c,d)] due to chaotization and Lyapunov divergence of classical trajectories is proportional to $e^{\lambda \tau}$ with the Lyapunov exponent $\lambda \propto \tau_F^{-1}$ [see app. B]. In the regime of small-angle scattering $\Delta \varphi_{sc} < \Delta \varphi_{\lambda}$, where $\Delta \varphi_{sc}$ denotes the typical scattering angle





Figure 7.2: Chaotic cavity in the 'classical' (a,c) and 'quantum' (b,d) regime. (a) In the classical case scattering is deterministic so that electrons with a specific momentum direction \mathbf{p}/p inside the cavity can be identified to originate either from the left or the right reservoir. This is illustrated by the random speckle pattern inside the cavity. (b) In the quantum case it cannot be decided whether the electrons came from the left or the right side because the wave function is spread over the whole cavity. (c,d) The shot noise power is determined by the fluctations $\langle f_C(1-f_C) \rangle$ of the state occupancy f_C . (c) In the classical limit f_C takes on either the value 1 or 0, and $S \propto \langle f_C(1-f_C) \rangle = 0$, although $\langle f_C \rangle \neq 0, 1$. If quantum diffraction occurs f_C can take on an arbitrary value between 0 and 1 within the energy interval $[\mu_R, \mu_L]$. In the 'full' quantum limit (strong diffraction) $\langle f_C(1-f_C) \rangle = \langle f_C \rangle (1-\langle f_C \rangle)$, and for $\langle f_C \rangle = 1/2$ the shot noise is $1/4 \cdot 2e|I|$.

7.2 Theory

Because of the relatively large cavities and small openings in the present experiments, $\tau_D \gg \tau_F$.

7.2.1 Classical regime

The classical regime is characterized by $\tau_Q \gg \tau_D$. In this regime classical (deterministic) trajectories are well defined. The distribution function f_C at a given point within the cavity and for a given energy E can be expressed as a function of momentum direction \mathbf{p}/p [Fig. 7.2(c)]. Since the cavity is chaotic, a large number of different trajectories cross this given point. However, classical physics is deterministic allowing to trace back each trajectory to its origin, which is uniquely determined to be the source contact (left contact at chemical potential μ_L) or the drain contact (right contact at chemical potential $\mu_R < \mu_L$). In the relevant energy window for transport, i.e. $\mu_L = \mu_R + eV \ge E \ge \mu_R$, f_C equals either 0 or 1. It is 1 if the particular trajectory originates from the left contact, while it is zero otherwise. The product $f_C(1-f_C)$, therefore, always equals zero and shot noise will be absent. Intuitively, this appears to be very surprising, because of the presence of chaos. Classical chaos leads to a very strange function f_C , which only takes on the values 0 and 1, but may switch between these two values in a very erratic and dense way. This, however, does not produce noise!

7.2.2 Quantum regime

If on the other hand, quantum diffraction cannot be neglected ($\tau_Q < \tau_D$), the situation is drastically changed. Due to quantum scattering on impurities and at the boundaries of the cavity an electron wave (classically the 'trajectory') may split into two or more partial waves leaving the cavity at different exits. A momentum state within the cavity cannot be traced back unambiguously to either contact, but carries information of both contacts simultaneously. Consequently, f_C is a weighted sum of f_L and f_R and can now take on values between 0 and 1. This uncertainty of not knowing where the electron came from and where it will go to is the source of noise. In the 'full' quantum limit, i.e. $\tau_Q \ll \tau_D$, the electron wave spreads over the whole cavity [see Fig. 7.2(d)]. and $\langle f_C(1 - f_C) \rangle = \langle f_C \rangle (1 - \langle f_C \rangle)$. If, in addition, the cavity is symmetric, $\langle f_C \rangle$ equals 1/2 yielding $S = 1/4 \cdot 2e|I|$ for the shot noise power [40, 60], what has only recently been confirmed [see chap. 6].

 $[\]Delta \varphi_{sc} \equiv |\mathbf{p} - \mathbf{p}'|/p$, the quantum scattering time τ_Q is determined by the Ehrenfest time τ_E of the chaotic system [111]. In this experiment however, quantum scattering is large-angle scattering and the quantum scattering time τ_Q is defined within the concept of semiclassical transport starting from the stationary Boltzmann equation for $f_{\mathbf{p}}$ as $\tau_Q^{-1} = \sum_{\mathbf{p}'} W_{\mathbf{pp}'}$, where $W_{\mathbf{pp}'}$ is the rate of quantum scattering [112] [see app. B].

7 Quantum-to-classical crossover in noise

7.2.3 Crossover between classical and quantum regime

The crossover between classical and quantum regimes in chaotic cavities was theoretically discussed by Agam *et al.* [111]. For a symmetric cavity the noise power S is [Eq. (6.24)]

$$S = S_{eq}[1 + F(\beta \coth \beta - 1)], \qquad \beta \equiv \frac{eV}{2k_B\theta}, \tag{7.2}$$

where $F \equiv S/2e|I|$ is the Fano factor defined as the current-normalized noise power at zero temperature. The Fano factor F decays to zero in the classical limit where $\tau_Q/\tau_D \to \infty$ and is to the first order in τ_Q/τ_D given by

$$F = \frac{1}{4} \left(1 - \frac{\tau_Q}{\tau_D} \right). \tag{7.3}$$

The noise power S can be written as $S = S_{Cl} + S_Q$, where $S_{Cl} = S_{eq} = 4k_B\theta/R$ is the equilibrium noise of the contacts. This part is insensitive to quantum mechanics and exists irrespectively of whether scattering within the cavity is quantum or classical. In contrast, the second non-equilibrium part $S_Q \propto F$ is a sensitive probe of quantum mechanics.

7.3 Experimental

Cavities of size L comparable to the mean free path l are experimentally realized in a two dimensional electron gas (2 DEG) [see also sect. 6.4 for experimental details]. Two QPCs, electrostatically defined by metallic split gates, connect the cavity to the reservoirs [Fig. 6.8(a) or inset of Fig. 7.3]. On the lateral two sides the electrons are confined by wet chemical etching of the 2 DEG. The openings of the cavity can be altered by varying the gate voltages. So called 'open' cavities are defined when both quantum point contacts are adjusted to a conductance plateau with an integer number Nof fully transmissive (T = 1) modes.

The deviations from the quantum limit (F = 1/4) towards the regime of deterministic scattering (F < 1/4), described by Eq. (7.3), is explored here by changing the dwell time τ_D , which is an experimentally controlable parameter. The dwell time depends on the area A of the cavity and the conductances $G_{L,R}$ of the contacts [Eq. (6.31)]:

$$\tau_D = \frac{2e^2m}{\pi\hbar^2} \cdot \frac{A}{(G_L + G_R)}.\tag{7.4}$$





Figure 7.3: Fano factor F = S/2e|I| vs inverse dwell time τ_D^{-1} of a symmetric cavity $(N_L = N_R)$. With increasing τ_D^{-1} the Fano factor F becomes smaller then 1/4 (quantum limit) indicating a crossover from full quantum to deterministic (classical) scattering inside the cavity. The numbers at the data points indicate the number of fully transmitted modes N at the contacts. (Top panel) Measured ratio between the total resistance R and the series resistance $R_L + R_R$ of the two contacts as a function of τ_D^{-1} . Since this ratio is always 1 the electronic motion inside the cavity is chaotic. (Inset) Scanning electron microscope picture of the cavity. For the zero-field data, gates A and B were used, whereas for the measurements in a magnetic field we worked with the gates A and C.

7.4 Zero-magnetic field measurements

In a first experiment the openings (conductances) of the contacts have been changed in order to alter τ_D . In Fig. 7.3, the Fano factor $F \equiv S/2e|I|$ of a symmetric $(N_L = N_R)$ cavity defined by the two gates A and B [Fig. 6.8(a)] is plotted as a function of the inverse dwell time τ_D^{-1} for four different settings $(N_L = N_R = 5, 14, 22 \text{ and } 40)$.

As the contacts are further opened and the dwell time is subsequently reduced, the shot noise is observed to decrease. The Fano factor F shows a pronounced decay below the quantum limit 1/4. A linear fit of the data to Eq. (7.3) yields a τ_{Q} of ~ 270 ps. It should be emphasized here, that the





Figure 7.4: A magnetic field reduces the dwell time $\tau_D \propto 1/B$. Theoretically, this effect can be described as a change of the area of the cavity, because for fields corresponding to a cyclotron radius R_c smaller than the size L of the cavity, an annulus is formed at the edge with an inner part of the cavity not contributing to transport and noise. For not too strong magnetic fields, this annulus is considered as a 'new' chaotic cavity with reduced area $A = 2R_cL_c$ with $L_c \sim L$ the circumference of the cavity.

total resistance R equals the series resistance $R_L + R_R$ of the two contacts within the measurement accuracy of ~ 3 % [top panel of Fig. 7.3]. Hence, the shot noise measurements are carried out in a regime where the direct (ballistic) transmission of electrons from the left to the right contact can be neglected. The suppression of shot noise observed here is a consequence of reduced diffraction and serves to demonstrate that shot noise disappears in the limit of purely classical scattering.

7.5 Magnetic field dependence

An alternative way to change the dwell time τ_D is to apply a perpendicular magnetic field. Since a magnetic field forces the electrons on circular orbits with the cyclotron radius $R_c = mv_F/eB$, the dwell time τ_D will be reduced with increasing magnetic field *B* provided $R_c < L$. An annulus of skipping orbits is formed in the vicinity of the cavity edge [inset Fig. 7.4], contributing both to transport and noise, whereas the bulk of the cavity does not participate in transport either because of very slow diffusion, or because of the energy gap between Landau levels at high magnetic fields. Such an annulus represents a 'new' chaotic cavity inside the actual cavity. For low magnetic fields (large filling factors), mthe electron dynamics inside the annulus can still be considered as being random (because of impurities or irregularities in the geometry of the cavity). Thus Eq. (7.2) and (7.3) are still valid with the area *A* in Eq. (7.4) replaced by $A = 2R_cL_c$, where $L_c \sim L$ is the circumference of the cavity. This leads to $1/\tau_D \propto B$.

Fig. 7.5 shows the measured Fano factor F as a function of inverse dwell time τ_D^{-1} in a magnetic field. Again a drastic reduction of F with increasing τ_D^{-1} is observed, while the total resistance R approximately equals the series resistance $R_L + R_R$ of the two contacts for B < 1.2 T [top panel in Fig. 7.5]. A linear fit according Eq. (7.3) results in a quantum scattering time τ_Q





Figure 7.5: (Top panel) Total resistance *R* normalized to the series resistance $R_L + R_R$ as a function of inverse dwell time τ_D^{-1} . (Bottom panel) Fano factor F = S/2e|I| vs τ_D^{-1} . *F* deviates from the quantum limit 1/4 with increasing τ_D^{-1} indicating a crossover from full quantum to classical scattering inside the cavity as for the measurements in zero magnetic field.

of ~ 485 ps, which is in qualitative agreement with τ_Q obtained from the measurements in zero field. If the magnetic field is further increased beyond 1.2 Tesla, the ratio $R/(R_L + R_R)$ starts to deviate from unity and equals 1/2 at the highest magnetic field. Here, we enter a new regime in which a significant fraction of electrons is ballistically transmitted from source to drain. The last measurement point in Fig. 7.5 with $F \simeq 0$ and $R/(R_L + R_R) = 0.5$ corresponds to the integer quantum Hall regime with filling factor $\nu = 4$, where the electrons propagate within two spin-degenerated ballistic edge states.

7.6 Conclusions

In conclusion, we have experimentally demonstrated that the shot noise in electron transport through mesoscopic conductors is a purely *quantum*

7 Quantum-to-classical crossover in noise

phenomenon. Shot noise is absent if scattering is completely deterministic. Chaotic cavities have proven to be an ideal model system for studying the quantum-to-classical crossover, because the electron dwell time can be changed either by varying the opening to the cavity or by applying a magnetic field. The observed suppression of shot noise demonstrates that Schottky's 'Schroteffekt' disappears in electrical conductors if quantum uncertainties are replaced by classical deterministic physics.

Chapter 8

Shot noise of series quantum point contacts

In this chapter the shot noise of a series of quantum point contacts forming a sequence of cavities in a two-dimensional electron gas is studied theoretically and experimentally. The noise in such a structure originates from local scattering at the point contacts as well as from chaotic motion of the electrons in the cavities [see chapt. 6 and 7]. For a single scatterer of transmission probability T the Fano factor F equals 1 - T and reaches 1/3 in the limiting case of an infinite number of scatterers [113]. Thus, the case of a large number of point contacts in series models a diffusive wire with randomly placed impurities, for which the Fano factor is 1/3, too [subsect. 2.4.2]. The work presented here is devoted to this crossover from F = 1 - T for a single scatterer to F = 1/3 in the diffusive regime.

8.1 Crossover from a single scatterer to the diffusive regime

The shot noise of a sequence of N planar tunnel barriers has been calculated by de Jong and Beenakker within a semiclassical description based on the Boltzmann-Langevin approach [113]. For equal transmission probabilities $T_{i=1,...,N} = T$ the Fano factor equals

$$F = \frac{1}{3} \left(1 + \frac{N(1-T)^2(2+T) - T^3}{[T+N(1-T)]^3} \right).$$
(8.1)

According to this result F reaches 1/3 with increasing barrier number $(N \to \infty)$ for any value of the transparency $T \in [0, 1]$.¹ Hence, in this

 $^{^{1}}$ In a simple one-dimensional system, in which electrons tunnel through N identical barriers, and where no correlations occur between current fluctuations at different barri-





Figure 8.1: Schematic of the considered system: N quantum point contacts forming a series of cavities. $f_{n=0,...,N}$ denote the distribution functions of the electrons. In such a system shot noise arises due to quantum diffraction inside the cavity as well as to partitioning at the contacts.

description a diffusive conductor can be modeled as the continuum limit of a series of tunnel barriers. However, the calculation in Ref. [113] is only valid for one-dimensional transport since it neglects transverse motion of the electrons in between the barriers². It is therefore not appropriate to describe our physical system consisting of a series of quantum point contacts. Here, we consider the case that there are cavities between the barriers in which the electrons scatter chaotically leading to additional cavity noise [40, 60].

In general, the fluctuations in the total current through a system as shown in Fig. 8.1 can be written as [60]

$$\delta I = \delta I_n^S + G_n (\delta V_{n-1} - \delta V_n), \quad n = 1, \dots, N$$
(8.2)

using the fact that the total current is conserved. $G_n = G_0 \sum_k T_{kn}$ ($G_0 = 2e^2/h$) is the conductance of the *n*-th barrier and δV_n the voltage fluctuations inside the *n*-th cavity. δI_n^S are the current fluctuations of a single QPC [34, 35] [see Eq. 2.32]:

$$\langle \delta I_n^S \delta I_m^S \rangle = S_n \delta_{nm} \tag{8.3}$$

ers, the noise of all barriers is just averaged. The total voltage noise in this case would be $S_V = N (2e|I|R^2)$ with R the resistance of a single barrier. Thus, the Fano factor follows as $F = S_V/(NR)^2/2e|I| = 1/N$ and shot noise disappears with increasing N. This is the case for example in a series of N vacuum tubes [114].

 $^{^{2}}$ A series of planar tunnel barriers could be modeled by series QPCs in the integer Quantum Hall regime with no backsacttering in the cavities between the barriers. Experimentally, it turned out to be very difficult to measure noise in this regime due to the non-linearities in the IV-characteristics of the point contacts at high magnetic fields [see also Fig. 2.6].

8.1 Crossover from a single scatterer to the diffusive regime

with

$$S_n = 2G_0 \sum_k \int_0^\infty dE \left[T_{kn} f_{n-1} (1 - f_{n-1}) + T_{kn} f_n (1 - f_n) + T_{kn} (1 - T_{kn}) (f_{n-1} - f_n)^2 \right].$$
(8.4)

Summing the square of Eq. (8.2) over n, while assuming equal conductances $G_n = G$, the total noise power follows as

$$S \equiv \langle \delta I^2 \rangle = \frac{1}{N^2} \sum_{n,m=1}^N \langle \delta I_n^S \delta I_m^S \rangle = \frac{1}{N^2} \sum_{n=1}^N S_n, \tag{8.5}$$

where we have assumed $\delta V_1 = \delta V_N = 0$, i.e. no fluctuations in the potential of the perfect metallic leads.

8.1.1 Non-interacting electrons

The distribution function f_n of the *n*-th cavity follows from the conservation of numbers of electrons in each energy interval [60] [see also sect. 6.3.1]

$$f_n(E) = \left(\frac{N-n}{N}\right) f_L(E) + \frac{n}{N} f_R(E), \qquad (8.6)$$

with $f_L(E) = f_F(E, eV_L, \theta)$ and $f_R(E) = f_F(E, eV_R, \theta)$ the equilibrium Fermi function in the left and right reservoir, respectively. θ denotes the bath temperature. For simplicity only one propagating mode will be considered for the moment with the backscattering parameter $\mathcal{R}_n \equiv 1 - T_n = \mathcal{R}$ assumed to be the same for all barriers. Substituting the distribution function f_n from Eq. (8.6) into Eq. (8.4), the total noise of N point contacts in series follows from Eq. (8.5):

$$S = \frac{4G_S k_B \theta}{N^2} \left\{ \frac{1}{3} \left[(2N^2 + 1) + (N^2 - 1) \frac{eV}{2k_B \theta} \coth\left(\frac{eV}{2k_B \theta}\right) \right] + \mathcal{R} \left[\frac{eV}{2k_B \theta} \coth\left(\frac{eV}{2k_B \theta}\right) - 1 \right] \right\}$$

$$(8.7)$$

 $G_S=G/N$ is the total conductance of the device. In the zero-temperature limit we obtain for the Fano factor

$$F \equiv \frac{S}{2e|I|} = \left(\frac{1}{3} - \frac{1}{3N^2} + \frac{\mathcal{R}}{N^2}\right).$$
 (8.8)

8 Shot noise of series quantum point contacts

Using the Fano factor F (8.8), $S_{eq} = 4G_S k_B \theta$ and $\beta \equiv (eV/2k_B \theta)$ [see Eq. (7.2)], Eq. (8.7) can be rewritten in a more simple and transparent form:

$$S = S_{eq} \left[\frac{2}{3} + \frac{1}{3} - \left(\frac{1}{3} - \frac{1}{3N^2} + \frac{\mathcal{R}}{N^2} \right) + \left(\frac{1}{3} - \frac{1}{3N^2} + \frac{\mathcal{R}}{N^2} \right) (\beta \coth \beta) \right]$$

= $S_{eq} \left[1 + F(\beta \coth \beta - 1) \right].$ (8.9)

For a single QPC (N = 1) F equals the backscattering parameter $\mathcal{R} = 1 - T$ as expected. For N = 2 we have a single cavity which is separated from the leads by two QPCs and for ideal contacts $(\mathcal{R} = 0)$ the Fano factor is 1/4. If the QPCs are in the tunneling regime $(\mathcal{R} \simeq 1)$ the noise is dominated by the QPCs and the dynamics inside the cavity play no role. The Poissonian voltage noise of the two contacts adds up resulting in a Fano factor 1/2. In the intermediate regime $F = \frac{1}{4}(1 + \mathcal{R})$. Increasing the number of QPCs $(N \to \infty)$ the Fano factor F reaches 1/3 *independent* of $\mathcal{R} = 1 - T$ as for the calculation in Ref. [113].

In Fig. 8.2(a) the result of Eq. (8.8) is compared to the result for onedimensional tunnel-barriers [113]. For point contacts with low transparencies (T = 0.1) the results are very similar because in this case the noise is dominated by the contacts. But for high transparencies (T = 0.9) the Fano factor including 'cavity noise' [Eq. (8.8)] increases much faster with the number of QPCs than the one-dimensional model of de Jong and Beenakker [113].

8.1.2 Interacting electrons

In case of electron-electron interaction within the cavities the distribution function inside the *n*-th cavity $f_n(E, \theta)$ equals a Fermi function $f_F(eV_n, \theta_n)$ at an elevated electron temperature θ_n [Fig. 8.3]. $V_n = V(1 - n/N)$ is the potential in the *n*-th cavity with $V = V_L - V_R$ the potential difference between left and right reservoir. The electron temperature θ_n follows from the energy balance equation using the Wiedemann-Franz law [107]:

$$\theta_n^2 = \theta^2 + \frac{3n(N-n)}{N^2} \left(\frac{eV}{\pi k_B}\right)^2.$$
 (8.10)

Using Eq. (8.4) and (8.5) the noise in this case of hot electrons is given by

$$S = \frac{2}{N^2} \sum_{n=1}^{N} \left\{ Gk_B(\theta_n + \theta_{n-1}) \right\}$$





Figure 8.2: Theoretical prediction for shot noise normalized to the Poissonian limit 2e|I| as a function of the number of QPCs N in series for cold (a) and hot electrons (b) at zero bath temperature. The crosses $(\times, +)$ in the left plot correspond to a calculation of de Jong and Beenakker [113] for one-dimensional transport, not including the noise arising from chaotic motion of the carriers in the cavities between the barriers. This is taken into account for Eq. (8.8).

$$+\sum_{k}\int_{0}^{\infty} dE \, T_{kn}(1-T_{kn}) \left[f_{n-1}-f_{n}\right]^{2} \bigg\}.$$
 (8.11)

The integral in Eq. 8.11 cannot be calculated analytically for the general case. Numerical results for the Fano factor are shown in Fig. 8.2(b)³. Analytical expressions can be given for $T_{i=1,...,N} = 1$

$$F = \frac{2\sqrt{3}}{\pi N^2} \sum_{n=1}^{N-1} \sqrt{n(N-n)}$$
(8.12)

and for the tunneling regime $(T_{i=1,\dots,N} \ll 1)$ in the diffusive limit $(N \to \infty)$

$$F = \frac{2\sqrt{3}}{\pi} \int_0^1 dx \sqrt{x(1-x)}$$
(8.13)

³The calculations were performed by a program written in C/C⁺⁺ [see app. D], which requires as input parameters the number of barriers N, the transmission probabilities $T_{1,2}$ of the first two modes (the others are 0), the current range, the bath temperature T as well as the Fermi energy of the 2 DEG.





Figure 8.3: Temperature profiles for 2, 3, 4 and 50 barriers in series. In case of inelastic electron-electron scattering electrons thermalize within the cavities and the temperature drastically increases above the bath temperature leading to enhanced noise. For a large number of barriers the temperature profile has a parabolic shape as for a metallic diffusive wire [115].

with $x = n/N \in [0, 1]$. In both cases, the Fanofactor F equals $\sqrt{3}/4$ in the continuum limit $N \to \infty$, in agreement with the result for a diffusive conductor with electron heating [41, 47, 116].

8.2 The device

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Experimentally, a structure as described in Fig. 8.1 has been realized in a 2 DEG by a series of up to four QPCs across a wet-chemical etched Hallbar. The 2 DEG forms at the interface of a standard GaAs/Al_{0.3}Ga_{0.7}Asheterojunction and its properties are described in sec. 6.4. The different split gates are spaced by 20 μ m and the Hallbar is 100 μ m width. Thus, the size of the cavity is rather large ($\tau_D \gg \tau_{ee}$) so that electron heating is present

8.3 Results and discussion



Figure 8.4: Scanning electron microscope images of the measured device. (a) For point contacts forming a sequence of large cavities. (b) The single point are defined with split gates.

[see sect. 6.5]. Voltage noise has been measured as a function of current for one to three QPCs in series⁴ with fixed transmission probabilities. The transmission of a single point contact can be adjusted by measuring its conductance $G = G_0 \sum_k T_k$ [for details see sect. 6.4]. The spectral density of the voltage fluctuations are typically averaged over a frequency bandwith of 1 kHz at around 7 to 9 kHz.

8.3 Results and discussion

In Fig. 8.5 the Fano factor $F \equiv S/2e|I|$ extracted from the shot noise measurements is plotted as a function of the number of point contacts. The black dots correspond to experimental data obtained for a transmission probability T = 0.9 of each single QPCs. The dashed line is the prediction of Eq. (8.8), whereas the crosses correspond to the one-dimensional calculation of Ref. [113]. For N = 1 shot noise is strongly suppressed compared to $S_{Poisson} = 2e|I|$, as expected for a single QPC with high transmission probability [36]. When N goes from 1 to 3 shot noise becomes larger. Because in the system depicted in Fig. 8.1 not only partition noise at the contacts contributes to the total noise but also additional cavity noise, the Fano factor indeed increases faster with increasing number of the contacts N than predicted by the one-dimensional theory. Since the cavities are large compared to the mean free path of the electrons the electrons stay relatively long in the cavity so that inelastical scattering cannot be excluded. However, the

⁴ In the experiment, we used only up to three QPCs because one of the four did not show proper conduction quantization.





Figure 8.5: The black points are experimental data for one mode transmitted at the point contacts with probability T = 0.9. The dashed line are theoretical predictions including cavity noise for non-interacting electrons [Eq. (8.8)]. The crosses correspond to the one-dimensional model of de Jong and Beenakker [113].

uncertainties of the experimental data in this experiment do not allow to distinguish between the cold- and hot-electron regime.

8.4 Conclusions

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We studied the shot noise of a series of QPCs forming a sequence of cavities. Theoretical calculations along the lines of Ref. [60] show that the shot noise reaches 1/3 and $\sqrt{3}/4$ of the Poissonian limit for hot and cold electrons, respectively, when the number of point contacts N is increased to infinity. Noise measurements on a series of 1, 2 and 3 QPCs defined in a 2 DEG are in reasonable qualitative agreement with a new theory that takes the contribution from the cavities into account.

Appendix A

Theoretical expressions for the fermionic HBT experiment

This appendix contains the derivation of the expressions for the auto- and cross-correlations between the fluctuations in the transmitted and reflected beam in the fermionic Hanbury Brown and Twiss experiment. In the first part the calculations are performed within the scattering approach [sect. 2.1] along the notes of Ref. [33]. In a second part we present a purely classical derivation of the same expressions.

A.1 Scattering approach

The current amplitudes of the incoming (\hat{a}) and outgoing states (\hat{b}) in the considered system [Fig. A.1] are related via the scattering matrix s:

$$\begin{pmatrix} \hat{b}_1\\ \hat{b}_2\\ \hat{b}_3 \end{pmatrix} = \begin{pmatrix} r_{11} & t_{21} & t_{31}\\ t_{12} & r_{11} & t_{32}\\ t_{13} & t_{23} & r_{33} \end{pmatrix} \begin{pmatrix} \hat{a}_1\\ \hat{a}_2\\ \hat{a}_3 \end{pmatrix}.$$
 (A.1)

Due to the chirality of the edge state the scattering matrix can be calculated easily. As an example, the transmission probability $|t_{32}|^2$ from contact 2



Figure A.1: Three probe geometry illustrating the experiment described in chapter 5. p and t are the transmission amplitudes of the two quantum point contacts. At $\nu = 2$ only one spin-degenerated edge channel is propagating.

A Theoretical expressions for the fermionic HBT experiment

to contact 3 is given by the probability for reflection at the first contact (1-p) times the probability for transmission at the second point contact t: $|t_{32}|^2 = (1-p)t$. For the complete scattering matrix we have

$$s = \begin{pmatrix} r_1 & 0 & t_1 \\ t'_1 t'_2 & r_2 & r'_1 t'_2 \\ t'_1 r'_2 & t_2 & r'_1 r'_2 \end{pmatrix}$$
(A.2)

with

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$$|t_1|^2 = |t_1'|^2 = p,$$
 $|r_1|^2 = |r_1'|^2 = 1 - p$ (A.3)

$$|t_2|^2 = |t'_2|^2 = t, \qquad |r_2|^2 = |r'_2|^2 = 1 - t.$$
 (A.4)

In the low-frequency limit the correlator between current fluctuations in the leads α and β expressed in terms of the scattering matrix is generally given by [33]

$$\langle \Delta I_{\alpha} \Delta I_{\beta} \rangle = 2 \frac{e^2}{h} \Delta \nu \sum_{\gamma \delta, \gamma \neq \delta} \int dE \operatorname{Tr} \left(\mathsf{s}_{\alpha \gamma}^{\dagger} \mathsf{s}_{\alpha \delta} \mathsf{s}_{\beta \delta}^{\dagger} \mathsf{s}_{\beta \gamma} \right) \\ \times f_{\gamma}(E) \left[1 - f_{\delta}(E) \right].$$
 (A.5)

Because of probability conservation the scattering matrix s must be unitary: $ss^{\dagger} = 1$. If time-reversal symmetry would hold it would be even symmetric. From the unitary property we use the following two relations to evaluate Eq. (A.5):

$$t_1' t_2' t_1'^* r_2'^* + r_2 t_2^* + r_1' t_2' r_1'^* r_2^* = 0 \Rightarrow t_2' r_2'^* = -r_2 t_2^*$$
 (A.6)

$$t_1' r_2' t_1'^* t_2'^* + t_2 r_2^* + r_1' r_2' r_1'^* t_2'^* = 0 \Rightarrow t_2 r_2^* = -r_2' t_2'^*.$$
 (A.7)

Furthermore, at zero temperature the Fermi-distribution functions $f_{\gamma,\delta}(E)$ in Eq. (A.5) are equal to the unitary step function $\theta(E - \mu_{\gamma,\delta})$. In this way we obtain for the cross-correlation of the current fluctuations at contacts 2 and 3

$$\langle \Delta I_2 \Delta I_3 \rangle = -2 \frac{e^2}{h} \Delta \nu \cdot t(1-t) \Big[p |\mu_1 - \mu_2| - p(1-p) |\mu_1 - \mu_3| + (1-p) |\mu_2 - \mu_3| \Big].$$
 (A.8)

Since contacts 2 and 3 are grounded, $\mu_2 = \mu_3 = 0$ [see Fig. A.1], so that¹

$$\langle \Delta I_2 \Delta I_3 \rangle = -2 \frac{e^2}{h} \mu_1 \Delta \nu \cdot t(1-t) p^2$$

¹In the experiment contacts 2 and 3 are connected to ground by two series resistor $R_S \ll R$ [see Fig. 5.4]. The current fluctuations in the transmitted and reflected beam

A.1 Scattering approach

$$= -2eI\,\Delta\nu \cdot t(1-t)p \tag{A.10}$$

with $I = p \cdot (e^2/h)V$ and $eV = \mu_1$. In the limit p = 1 we obtain $\langle \Delta I_2 \Delta I_3 \rangle = -2e|I| \Delta \nu \cdot t(1-t)$. The auto-correlations are calculated in the same way as the cross-correlation:

$$\langle \Delta I_1^2 \rangle = 2 \frac{e^2}{h} \Delta \nu \cdot p(1-p) |\mu_1 - \mu_3|$$

$$\langle \Delta I_2^2 \rangle = 2 \frac{e^2}{h} \Delta \nu \cdot \{ pt(1-t) |\mu_1 - \mu_2|$$

$$(A.11)$$

$$\Delta I_2^2 \rangle = 2 \frac{1}{h} \Delta \nu \cdot \{ pt(1-t) | \mu_1 - \mu_2 | + p(1-p)t^2 | \mu_1 - \mu_3 | + (1-p)t(1-t) | \mu_2 - \mu_3 | \}$$
(A.12)

$$\langle \Delta I_3^2 \rangle = 2 \frac{e^2}{h} \Delta \nu \cdot \left\{ pt(1-t) |\mu_1 - \mu_2| + p(1-p)(1-t)^2 |\mu_1 - \mu_3| + (1-p)t(1-t) |\mu_2 - \mu_3| \right\}.$$
 (A.13)

In summary, the spectral density of the cross- and auto-correlations $S_{nm} = \langle \Delta I_n \Delta I_m \rangle_{\omega}$ for $\mu_2 = \mu_3 = 0$ are [45]

$$\frac{S}{2e|I|} = \begin{pmatrix} 1-p & -t(1-p) & -(1-t)(1-p) \\ -t(1-p) & t(1-pt) & -t(1-t)p \\ -(1-t)(1-p) & -t(1-t)p & (1-t)\left[1-(1-t)p\right] \end{pmatrix}.$$
 (A.14)

 $S_{nm}\leq 0$ for $n\neq m$ and ≥ 0 for n=m. In the Poissonian limit $(p\to 0)$ and for a beam splitter with t=1/2 we obtain

$$F \equiv \frac{S}{2e|I|} = \begin{pmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1/2 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix},$$
(A.15)

whereas in the case that the states of the incident beam are fully occupied (p = 1) the Fano factors F are given by (t = 1/2)

$$F \equiv \frac{S}{2e|I|} = \begin{pmatrix} 0 & 0 & 0\\ 0 & 1/4 & -1/4\\ 0 & -1/4 & 1/4 \end{pmatrix}.$$
 (A.16)

$$\langle \Delta I_2 \Delta I_3 \rangle = \frac{\langle \Delta V_2 \Delta V_3 \rangle}{R_S^2} \cdot \left[\frac{(1+2xt)(1+xt)^2}{1-x(1-t)} \right]. \tag{A.9}$$

are detected via the voltage fluctuations across these series resistor R_S . Consequently the chemical potentials μ_2 and μ_3 are fluctuating and a correction of the order $x \equiv R_S/R$ has to be applied to Eq. (A.10). For p = 1 the cross-correlation of the current fluctuations is related to the cross-correlation of the voltage fluctuations by

For $\nu = 1$, i.e. only one spin-degenerated edge channel, $x \simeq 12.9^{-1}$, and for t = 1/2 the correction factor [...] equals $\simeq 1.2$.

A Theoretical expressions for the fermionic HBT experiment

A.2 Classical derivation

The expressions S_{22} and S_{23} in Eq. (A.14) for the cross- and auto-correlation of the fluctuations in the transmitted and reflected beam can also be derived in terms of classical physics. Consider a stream of hard core particles incident on a beam splitter with n particles arriving at the beam splitter per unit time interval. Treating electrons as hard core particles somehow incorporates the fact that electrons are fermions which cannot occupy the same state twice [117]. The average particle number $\langle n \rangle$ is given by $\sum_{n=0}^{\infty} \rho_n n$ where ρ_n denotes the probability distribution for exactly *n* incident particles. At the beam splitter the particles are scattered either to the transmitted channel with probability t or to the reflected with probability (1-t) with t the transparency of the beam splitter. For the mean transmitted current we have $\langle n_t \rangle = t \langle n \rangle$ and for the mean reflected current $\langle n_r \rangle = (1-t) \langle n \rangle$. In order to calculate the correlator of the fluctuations $\langle \Delta n_t^2 \rangle = \langle n_t^2 \rangle - \langle n_t \rangle^2$ of n_t around its mean value $\langle n_t \rangle$ we will next evaluate the average of the squared transmitted current $\langle n_t^2 \rangle$. The probability that *i* particles out of n will be transmitted while the others are reflected is given by the factor $t^{i}(1-t)^{n-i}$. Furthermore, we can choose *i* of *n* indistinguishable particles in $n!/[i!(n-i)!] \equiv \binom{n}{i}$ different ways. Thus, the average over the transmitted current squared yields

$$\langle n_t^2 \rangle = \sum_n^\infty \rho_n \left\{ \sum_{i=0}^n \binom{n}{i} t^i (1-t)^{n-i} \cdot i^2 \right\}$$

= $t \langle n \rangle + t^2 \langle n(n-1) \rangle.$ (A.17)

Here, we used the relation² $\sum_{i=0}^{n} {n \choose i} t^{i} (1-t)^{n-i} \cdot i^{2} = nt + n(n-1)t^{2}$. From Eq. (A.17) the auto-correlation between the current fluctuations in

²Proof: Consider the binomial expression $(a+b)^n = \sum_{i=0}^n {n \choose i} a^{n-i} b^i$ with (a+b) = 1. Taking the first and second derivative with respect to b we obtain:

$$n(a+b)^{n-1} = n = \sum_{i=1}^{n} {n \choose i} a^{n-i} b^{i-1} \cdot i$$
 (A.18)

$$n(n-1)(a+b)^{n-2} = n(n-1) = \sum {\binom{n}{i}} a^{n-i} b^{i-2} \cdot i(1-i).$$
(A.19)

Finally, the last equation can be rewritten in the following way:

$$n(n-1) = \left[\sum {\binom{n}{i}} a^{n-i} b^i \cdot i^2\right] b^{-2} - \underbrace{\left[\sum {\binom{n}{i}} a^{n-i} b^{i-1} \cdot i\right]}_{=n} b^{-1}(A.20)$$

$$n(n-1)b^{2} = \sum_{i=1}^{n} {n \choose i} a^{n-i}b^{i} \cdot i^{2} - nb$$
(A.21)

$$nb + n(n-1)b^2 = \sum {n \choose i} a^{n-i} b^i \cdot i^2.$$
 (A.22)

A.2 Classical derivation

the transmitted beam follows as

$$\begin{split} \langle \Delta n_t^2 \rangle &\equiv \langle n_t^2 \rangle - \langle n_t \rangle^2 \\ &= t \langle n \rangle + t^2 \underbrace{\{ \langle n^2 \rangle - \langle n \rangle^2 \}}_{= \langle \Delta n^2 \rangle} - t^2 \langle n \rangle \\ &= t(1-t) \cdot \underbrace{\{ \frac{t}{1-t} \langle \Delta n^2 \rangle + \langle n \rangle \}}_{\text{(A.23)}}. \end{split}$$

Similar the cross-correlation between fluctuations in the transmitted and reflected beam can be calculated. We first derive the average of the product $n_t n_r$ of the current in the transmitted and reflected beam:

$$\langle n_t n_r \rangle = \sum_{n=1}^{\infty} \rho_n \left\{ \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} \cdot i(n-i) \right\}$$

$$= -t \langle n \rangle - t^2 \langle n(n-1) \rangle + \sum_{n=1}^{\infty} \rho_n n \underbrace{\left\{ \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} \cdot i \right\}}_{= nt}$$

$$= -t \langle n \rangle - t^2 \langle n(n-1) \rangle + t \langle n^2 \rangle$$

$$= t(1-t) \left\{ \langle n^2 \rangle - \langle n \rangle \right\}$$
(A.24)

The cross-correlation is then given by

$$\begin{split} \langle \Delta n_t \Delta n_r \rangle &= \langle n_t n_r \rangle - \langle n_t \rangle \langle n_r \rangle \\ &= t(1-t) \left\{ \langle n^2 \rangle - \langle n \rangle^2 3 \right\} - \langle n \rangle^2 t(1-t) \\ &= t(1-t) \cdot \left\{ \langle \Delta n^2 \rangle - \langle n \rangle \right\}. \end{split}$$
 (A.25)

Finally, we briefly show that the formulas Eq. (A.23) and Eq. (A.25) are equivalent to the expressions of Eq. (A.14). In case of an additional barrier with transparency p in front of the beam splitter $\langle n \rangle = p \langle n_{ex} \rangle$ where $\langle n_{ex} \rangle$ corresponds to the externally applied current. The fluctuations in the incident beam are $\langle \Delta n^2 \rangle = p(1-p) \langle n_{ex} \rangle$. For the normalized correlators we obtain

$$\frac{\langle \Delta n_t^2 \rangle}{\langle n \rangle} = t(1 - pt) \tag{A.26}$$

$$\frac{\langle \Delta n_t \Delta n_r \rangle}{\langle n \rangle} = -t(1-t)p, \qquad (A.27)$$

which are the same expression as the ones derived before with the scattering approach.

A Theoretical expressions for the fermionic HBT experiment
Appendix B

Semiclassical theory of noise in chaotic cavities

Besides a full quantum mechanical description of transport and noise in mesoscopic systems a semiclassical theory has been introduced, too. In the following such a description of noise in chaotic cavities is summarized closely following Ref. [112].

For $\lambda_F \ll l$ transport and noise in electronic systems can be described by 'classical' single particle distribution functions $f_{\mathbf{p}}(\mathbf{r},t)$ and two-particle correlation functions

$$\mathcal{F}_{\mathbf{pp}'}(\mathbf{rr}', t) = \langle \delta f_{\mathbf{p}}(\mathbf{r}, t) \delta f_{\mathbf{p}'}(\mathbf{r}', 0) \rangle.$$
(B.1)

The fluctuating distribution function $f_{\mathbf{p}}(\mathbf{r},t)$ equals $(2\pi)^d$ times the density of electrons¹ with position \mathbf{r} and momentum \mathbf{p} at time t [103, 113]. $\delta f(t) \equiv f(t) - \langle f(t) \rangle$ is the time dependent deviation from the time-averaged distribution function $\langle f_{\mathbf{p}}(\mathbf{r},t) \rangle \equiv f_{\mathbf{p}}(\mathbf{r})$. $f_{\mathbf{p}}$ itself obeys the stationary Boltzmann equation

$$\frac{d}{dt}f_{\mathbf{p}}(\mathbf{r}) \equiv \mathbf{v}\nabla f_{\mathbf{p}}(\mathbf{r}) = -I_Q[f_{\mathbf{p}}(\mathbf{r})], \qquad (B.2)$$

where the classical dynamics of the electrons (such as the motion in a smooth atomic potential) is taken into account on the left hand side of this equation (i.e. via the dispersion relations), while *quantum scattering* is described by the collision integral

$$I_Q[f_{\mathbf{p}}(\mathbf{r})] = \sum_{\mathbf{p}'} W_{\mathbf{p}\mathbf{p}'} [f_{\mathbf{p}}(\mathbf{r}) - f_{\mathbf{p}'}(\mathbf{r})].$$
(B.3)

The kernel $W_{\mathbf{pp}'} = W_{\mathbf{p'p}}$ is the transition rate for scattering from the momentum state **p** to **p**'. The correlation function $\mathcal{F}_{\mathbf{pp}'}$ satisfies the time-

 $^{^{1}}d$ denotes the dimension of the system.

B Semiclassical theory of noise in chaotic cavities

dependent Boltzmann equation in the variables \mathbf{r} , \mathbf{p} , t [103, 113]

$$\left(\frac{d}{dt} + I_Q\right) \mathcal{F}_{\mathbf{pp}'}(\mathbf{rr}', t) = 0 \tag{B.4}$$

with the equal-time correlation given by

$$\mathcal{F}_{\mathbf{p}\mathbf{p}'}(\mathbf{r}\mathbf{r}',0) = \delta(\mathbf{r}-\mathbf{r}')\delta(\mathbf{p}-\mathbf{p}') f_{\mathbf{p}'}(\mathbf{r}) \left[1-f_{\mathbf{p}}(\mathbf{r})\right].$$
(B.5)

Classically, I_Q is zero and $f_{\mathbf{p}}(\mathbf{r})$ is constant along the classical trajectory, so that the stationary Boltzmann equation Eq. (B.2) can be solved easily. In this case, the electronic motion is completely deterministic and one knows from which lead an electron originates. Quantum diffraction leads to $I_Q \neq 0$ contributing both to noise and resistance (backscattering).

In diffusive systems quantum scattering dominates resistance and noise. This is completely different for chaotic cavities where the voltage drop occurs only across the contacts determining the resistance. Because electrons transmit an open quantum point contact with unit probability, no noise is generated at the contacts [see sect. 7.2]. Because the dwell time is a free parameter in the experiment [sect. 7.3] chaotic cavities can be used as a model system to study the fundamental origin of shot noise as discussed in chap. 7.

According to Ref. [60] the noise power of a chaotic cavity is determined by the cavity noise temperature θ_C and the bath temperature θ :

$$S_{nm} = -2k_B G_{nm}(\theta + \theta_C)$$
 with (B.6)

$$k_B \theta_C = \int dE \langle f_{\mathbf{p}}(\mathbf{r})[1 - f_{\mathbf{p}}(\mathbf{r})] \rangle,$$
 (B.7)

where $\langle \ldots \rangle = \sum_{p} \delta(E - E_{\mathbf{p}})\nu^{-1}(\ldots)$ denotes the average over momentum direction \mathbf{p}/p with $\nu = dN/dE$ the density of states. The delta function $\delta(E - E_{\mathbf{p}})$ ensures that only these energy states are counted with the energy corresponding to the momentum state \mathbf{p} . G_{nm} in Eq. (B.6) denotes the multiterminal conductance [Eq. (6.9)].

As we already discussed in chap. 7 the distribution function $f_{\mathbf{p}}(\mathbf{r})$ of a chaotic cavity $(\tau_D \gg \tau_F)$ is a random function of the momentum direction \mathbf{p}/p because slight initial variations in \mathbf{p}/p are exponentially amplified as time evolves [Fig. B.1]. The scale of these fluctuations in $f_{\mathbf{p}}(\mathbf{r})$ cavity is given by the 'correlation angle' $\Delta \varphi_{\lambda}(\tau) \sim \exp(-\lambda \tau) \sim \exp(-\tau_D/\tau_F)$ with the Lyapunov exponent $\lambda \sim 1/\tau_F$ of the underlying classical chaotic system [111, 118]. Comparing $\Delta \varphi_{\lambda}$ to the typical scattering angle $\Delta \varphi_{sc} \equiv |\mathbf{p} - \mathbf{p}'|/p$ [see Fig. B.1] two different regimes can be distinguished [118]:

- (i) quantum chaos: $\Delta \varphi_{sc} < \Delta \varphi_{\lambda}$, and
- (ii) quantum disorder: $\Delta \varphi_{sc} > \Delta \varphi_{\lambda}$.

B.1 'Quantum disorder'



Figure B.1: The distribution function within a chaotic cavity [see also Fig. 7.2(c,d)] is a random function of the momentum direction \mathbf{p}/p . $\Delta \varphi_{\lambda}$ is the 'correlation length' of these fluctuations and $\Delta \varphi_{sc}$ denotes the typical scattering angle.

The scattering anlge $\Delta \varphi_{sc}$ depends on the characteristic scale *a* of the scattering potential: after interaction of an electron with a scatterer of size *a*, the quantum uncertainty in the direction of its momentum is of the order λ_F/a . Thus a lower-bound estimate for $\Delta \varphi_{sc}$ is $1/(Lk_F)$ with *L* the size of the cavity. For the cavities used in the experiments discussed in chap. 6 and 7 $\Delta \varphi_{sc} \simeq 10^{-2}$ and $\Delta \varphi_{\lambda} \simeq 10^{-5}$. We think that the quantum scattering is due to a few short-range impurities within the cavity, and it is therefore large-angle scattering. This would be also consistent with the observation that the mean free path in the 2 DEG is of the same order as the cavity size. Thus the cavities are in the regime of 'quantum disorder'.

B.1 'Quantum disorder'

In order to calculate the noise power the cavity noise temperature θ_C of Eq. (B.7) has to be determined. Because electrons are scattered over large angles compared to $\Delta \varphi_{\lambda}$ the sum over \mathbf{p}' in Eq. (B.3) can be considered as

B Semiclassical theory of noise in chaotic cavities



Figure B.2: At the contact L_n of the *n*-th reservoir the distribution function $f_{\mathbf{p}}$ is either f_n , when the electron stems from the reservoir, or f_C , when it comes from the cavity. If $f_{\mathbf{p}} = f_n$ for a certain \mathbf{p} , then $f_{-\mathbf{p}} = f_C$.

'course graining' over the rapid fluctuations in $f_{\mathbf{p}}$, and hence the distribution function $f_{\mathbf{p}}$ can be replaced by the averaged distribution of the cavity: $f_{\mathbf{p}} \rightarrow \langle f \rangle \equiv f_C$. Thus Eq. (B.2) can be rewritten as

$$\mathbf{v}\nabla f_{\mathbf{p}}(\mathbf{r}) + \tau_Q^{-1}[f_{\mathbf{p}}(\mathbf{r}) - f_C] = 0, \qquad (B.8)$$

where τ_Q is the quantum scattering time defined via $\tau_Q^{-1} \equiv \sum_{\mathbf{p}'} W_{\mathbf{pp}'}$. Multiplying Eq. (B.8) by $\delta(E - E_p)$ and summing over \mathbf{p} we arrive at

$$\sum_{\mathbf{p}} \delta(E - E_{\mathbf{p}}) \mathbf{v} \nabla f_{\mathbf{p}} = \tau_Q^{-1} \sum_{\mathbf{p}} \delta(E - E_{\mathbf{p}}) [f_C - f_{\mathbf{p}}]$$

$$\nu \langle \mathbf{v} \nabla f_{\mathbf{p}} \rangle = \nu \tau_Q^{-1} \underbrace{[f_C - \langle f_{\mathbf{p}} \rangle]}_{= 0} \quad (\langle f_{\mathbf{p}} \rangle = f_C)$$

$$\implies \nabla \mathbf{j}(E, \mathbf{r}) = 0 \quad \text{with} \qquad (B.9)$$

$$\mathbf{j}(E, \mathbf{r}) \equiv 2e\nu \langle \mathbf{v} f_{\mathbf{p}} \rangle. \qquad (B.10)$$

Here, we used in the second last step $\nabla \mathbf{v} = 0$. Integration of Eq. (B.9) over the area A of the cavity using Gauss' theorem gives the current conservation for any given energy interval:

$$\iint_{A} d\mathbf{r} \,\nabla \mathbf{j}(E, \mathbf{r}) = \oint_{\partial A} dl \,\mathbf{n} \cdot \mathbf{j}(E, \mathbf{r}) = 0$$

$$\implies \sum_{n} J_{n}(E) = 0 \quad \text{with} \tag{B.11}$$

$$J_n(E) = 2e\nu \int_{L_n} dl \, \langle \mathbf{nv} f_{\mathbf{p}} \rangle. \tag{B.12}$$

 L_n denotes the opening of the *n*-th contact and **n** is the outward normal vector to the surface [see Fig. B.2].

The next step is to express the current J_n in terms of the distribution functions inside the cavity f_C and in the reservoirs f_n . At the contact L_n the distribution function $f_{\mathbf{p}}$ equals either f_C or f_n depending on whether an electron originates from the cavity or from the reservoir [Fig. B.2]. For example if $f_{\mathbf{p}} = f_n$ for a certain momentum \mathbf{p} , then obviously $f_{-\mathbf{p}} = f_C$. Let us now introduce the function $\eta_p = \pm 1$ which equals +1 for electrons

B.1 'Quantum disorder'

stemming from the cavity and -1 for electrons from the reservoir. Assume it is for sure that the electron passing L_n with momentum \mathbf{p}' originates from the cavity. In this case $f_{\mathbf{p}'} = f_C$, $f_{-\mathbf{p}'} = f_n$ and $\mathbf{v}_{-\mathbf{p}'} = -\mathbf{v}_{\mathbf{p}'}$, so that one can write

$$\begin{aligned} \mathbf{v}_{\mathbf{p}'} f_{\mathbf{p}'} + \mathbf{v}_{-\mathbf{p}'} f_{-\mathbf{p}'} &= \mathbf{v}_{\mathbf{p}'} \left(f_C - f_n \right) \\ &= 1/2 \left[\mathbf{v}_{\mathbf{p}'} (f_C - f_n) + \mathbf{v}_{\mathbf{p}'} (f_C - f_n) \right] \\ &= 1/2 \left[\mathbf{v}_{\mathbf{p}'} (f_C - f_n) - \mathbf{v}_{-\mathbf{p}'} (f_C - f_n) \right] \\ &= 1/2 \left[\mathbf{v}_{\mathbf{p}'} \eta_{\mathbf{p}'} + \mathbf{v}_{-\mathbf{p}'} \eta_{-\mathbf{p}'} \right] (f_C - f_n). \end{aligned}$$

Thus we have

$$\langle \mathbf{n} \mathbf{v} f_{\mathbf{p}} \rangle = \sum_{\mathbf{p}} \delta(E - E_{\mathbf{p}}) \nu^{-1} \mathbf{n} \mathbf{v} f_{\mathbf{p}}$$

$$= 1/2 \sum_{\mathbf{p}} \delta(E - E_{\mathbf{p}}) \nu^{-1} \mathbf{n} \mathbf{v} \eta_{\mathbf{p}} (f_C - f_n)$$

$$= 1/2 \langle \mathbf{n} \mathbf{v} \eta_{\mathbf{p}} \rangle (f_C - f_n)$$
(B.13)

and the current J_n can be expressed as

$$J_n = e^{-1} G_n (f_C - f_n) \quad \text{with} \quad G_n \equiv e^2 \nu \int_{L_n} dl \, \langle \mathbf{n} \mathbf{v} \eta_{\mathbf{p}} \rangle. \tag{B.14}$$

From Eq. (B.11) and (B.14) the distribution function inside the cavity f_C is obtained as [Eq. (6.14)]

$$f_C = \sum_n \alpha_n f_n(E)$$
 with $\alpha_n \equiv G_n / \sum_n G_n.$ (B.15)

 f_C does not depend on τ_Q because the small contribution to the momentum relaxation due to quantum scattering is neglected.

In order to obtain the cavity noise which is proportional to $\langle f_{\mathbf{p}}(1-f_{\mathbf{p}})\rangle = f_C - \langle f_{\mathbf{p}}^2 \rangle$ we now express $\langle f_{\mathbf{p}}^2 \rangle$ in terms of f_n and f_C as we did before for the current J_n . Multiplying Eq. (B.8) with $f_{\mathbf{p}}$ and averaging over \mathbf{p}/p yields

$$\nabla \langle \mathbf{v} f_{\mathbf{p}}^2 \rangle = -2\tau_Q^{-1} [\langle f_{\mathbf{p}}^2 \rangle - f_C^2]. \tag{B.16}$$

Here, we used $\langle f_{\mathbf{p}} \rangle = f_C$ and $\nabla(\mathbf{v}f_{\mathbf{p}}^2) = (\nabla \mathbf{v})f_{\mathbf{p}}^2 + 2f_{\mathbf{p}}\mathbf{v}(\nabla f_{\mathbf{p}}) = 2f_{\mathbf{p}}\mathbf{v}(\nabla f_{\mathbf{p}})$ since $\nabla \mathbf{v} = 0$. As for Eq. (B.9) we integrate Eq. (B.16) over the area A of the cavity using again the η -function, which distinguishes electrons comming from the cavity or from the reservoir. For the left hand side of Eq. (B.16) we obtain

$$\iint_{A} d\mathbf{r} \,\nabla \langle \mathbf{v} f_{\mathbf{p}}^{2} \rangle \quad = \quad \sum_{n} \int_{L_{n}} dl \,\langle \mathbf{n} \mathbf{v} f_{\mathbf{p}}^{2} \rangle$$

B Semiclassical theory of noise in chaotic cavities

$$= \frac{1}{2} \sum_{n} \left[\langle f_{\mathbf{p}}^{2} \rangle - f_{n}^{2} \right] \int_{L_{n}} dl \, \langle \mathbf{n} \mathbf{v} \eta_{\mathbf{p}} \rangle$$
$$= \frac{1}{2e^{2}\nu} \sum_{n} \left[\langle f_{\mathbf{p}}^{2} \rangle - f_{n}^{2} \right] G_{n}. \tag{B.17}$$

In the first step we have used here $\langle \mathbf{nv} f_{\mathbf{p}}^2 \rangle = \langle \mathbf{nv} \rangle \langle f_{\mathbf{p}}^2 \rangle$, because the distribution function $f_{\mathbf{p}}$ fluctuates on a much smaller scale than the typical scattering angle ('coarse graining'). The term on the right hand side of Eq. (B.16) is simply multiplied by the area A, so that we arrive at

$$\frac{1}{2e^2\nu}\sum_{n}\left[\langle f_{\mathbf{p}}^2\rangle - f_n^2\right]G_n = -\frac{2A}{\tau_Q}\left[\langle f_{\mathbf{p}}^2\rangle - f_C^2\right].$$
 (B.18)

Next, we multiply with $2e^2\nu$ and divide by $\sum_n G_n$:

$$\langle f_{\mathbf{p}}^2 \rangle - \sum_n G_n f_n^2 / \sum_n G_n = -\tau_Q^{-1} \Big(4e^2 \nu A / \sum_n G_n \Big) \left[\langle f_{\mathbf{p}}^2 \rangle - f_C^2 \right]$$

$$\langle f_{\mathbf{p}}^2 \rangle - \sum_n \alpha_n f_n^2 = -\frac{\tau_D}{\tau_Q} \left[\langle f_{\mathbf{p}}^2 \rangle - f_C^2 \right],$$
(B.19)

where we have defined the dwell time τ_D as

$$\tau_D \equiv 4e^2 \nu A / \sum_n G_n. \tag{B.20}$$

Solving Eq. (B.19) for $\langle f_{\mathbf{p}}^2\rangle$ we obtain

$$\langle f_{\mathbf{p}}^2 \rangle = \frac{\tau_D}{\tau_D + \tau_Q} f_C^2 + \frac{\tau_Q}{\tau_D + \tau_Q} \sum_n \alpha_n f_n^2$$

$$= \frac{\tau_D}{\tau_D + \tau_Q} \left(f_C^2 - \sum_n \alpha_n f_n^2 \right) + \sum_n \alpha_n f_n^2.$$
(B.21)

Thus

$$\begin{aligned} \langle f_{\mathbf{p}}(1-f_{\mathbf{p}}) \rangle &= \langle f_{\mathbf{p}} \rangle - \langle f_{\mathbf{p}}^2 \rangle = f_C - \langle f_{\mathbf{p}}^2 \rangle \\ &= \sum_n \alpha_n f_n - \frac{\tau_D}{\tau_D + \tau_Q} \left(f_C^2 - \sum_n \alpha_n f_n^2 \right) - \sum_n \alpha_n f_n^2 \\ &= \frac{\tau_D}{\tau_D + \tau_Q} \left(\sum_n \alpha_n f_n^2 - f_C^2 \right) + \sum_n \alpha_n f_n (1-f_n). \end{aligned}$$

Since

$$\sum_{n} \alpha_{n} f_{n}^{2} - f_{C}^{2} = \frac{1}{2} \sum_{n} \alpha_{n} f_{n}^{2} + \frac{1}{2} \sum_{n} \alpha_{n} f_{n}^{2} - \left(\sum_{n} \alpha_{n} f_{n}\right)^{2}$$

B.2 'Quantum chaos'

$$= \frac{1}{2} \sum_{n,m} \alpha_n \alpha_m f_m^2 + \frac{1}{2} \sum_{n,m} \alpha_n \alpha_m f_n^2 - \sum_{n,m} \alpha_n \alpha_m f_n f_m$$
$$= \frac{1}{2} \sum_{n,m} \alpha_n \alpha_m (f_n - f_m)^2, \qquad (B.22)$$

where we used $\sum_{n} \alpha_n = 1$, we finally arrive at

$$\langle f_{\mathbf{p}}(1-f_{\mathbf{p}})\rangle = \sum_{n} \alpha_{n} f_{n}(1-f_{n}) + \frac{\tau_{D}}{\tau_{D}+\tau_{Q}} \frac{1}{2} \sum_{n,m} \alpha_{n} \alpha_{m} (f_{n}-f_{m})^{2}.$$
(B.23)

Integrating over energy E according to Eq. (B.7) we obtain the cavity noise temperature

$$\theta_C = \theta + \frac{\tau_D}{\tau_D + \tau_Q} \mathcal{T}, \qquad (B.24)$$

$$\mathcal{T} = \frac{1}{2} \sum_{n,m} \alpha_n \alpha_m \int dE \, (f_n - f_m)^2. \tag{B.25}$$

In the particular case of only two terminals the noise power S and the Fano factor F follow as

$$S = S_{eq}[1 + F(\beta \coth \beta - 1)], \quad \beta = \frac{eV}{2k_B\theta}, \quad (B.26)$$

$$F = \frac{\tau_D}{\tau_D + \tau_Q} \frac{G_1 G_2}{(G_1 + G_2)^2},$$
 (B.27)

where $S_{eq} = 4Gk_B\theta$ with $G = G_1G_2/(G_1 + G_2)$ the total conductance. For a symmetric cavity $(G_1 = G_2)$ the Fano factor thus equals 1/4 in the quantum regime $(\tau_D \gg \tau_Q)$ and vanishes in the classical regime where $\tau_Q \sim 1/\hbar \to \infty$.

B.2 'Quantum chaos'

If $\Delta \varphi_{sc} < \Delta \varphi_{\lambda}$ the 'coarse graining' $f_{\mathbf{p}} \rightarrow \langle f \rangle \equiv f_C$ of Eq. (B.8) is not valid. This regime of 'quantum chaos', where $f_{\mathbf{p}} \equiv f(\varphi)$ is a smooth function on the scale $\Delta \varphi_{sc}$, was considered in a recent paper by Agam *et al.* [111]. They obtained for the Fano factor of a symmetric cavity

$$F = \frac{1}{4} \exp\left(-\frac{\tau_E}{\tau_D}\right),\tag{B.28}$$





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Figure B.3: Quantum-to-classical crossover of the Fano factor for the regime of 'quantum disorder' [Eq. (B.27)], 'quantum chaos' [Eq. (B.28)] and the 1st-order expansion of both predictions [Eq. (7.3)]. The gray area roughly denotes the regime in which shot noise has been measured in the experiments discussed in chap. 7 in order to extract the quantum scattering time τ_Q .

with $\tau_E = 1/\lambda \ln(a/\lambda_F) \sim \tau_F \ln(k_F L)$ the so called Ehrenfest time. Because $\tau_E \ll \tau_D$ in the experiment the exponential in Eq. (B.28) can be expanded:

$$F = \frac{1}{4} \left(1 - \frac{\tau_E}{\tau_D} \right). \tag{B.29}$$

This result coincides with the result of Eq. (B.27) $(G_1 = G_2)$ in the limit $\tau_Q \ll \tau_D$ if $\tau_Q \simeq \tau_E$. The two different expressions for the crossover of shot noise from the 'quantum' to the 'classical' regime for 'qantum chaos' [Eq. (B.28)] and 'quantum disorder' [Eq. (B.27)] are compared in Fig. B.3.

Appendix C

Shot noise of vacuum tubes

In this appendix the shot noise of vacuum tubes is discussed in detail. It will be shown that the 'Schroteffekt' in vacuum tubes, first described by Schottky in 1918 [26], is a classical phenomenon which is due to thermal fluctuations of the occupancy in the cathode. Some parts of this appendix are based on notes by C. Schönenberger ("Shot Noise in Schottky's vacuum tube is CLASSICAL", ITP, UCSB (2001)) originating from discussions with E. V. Sukhorukov, C. W. J. Beenakker, H. Grabert and myself.

C.1 Shot noise of a two-terminal conductor

We start with a simple derivation of the expression for the power spectral density of the noise of a two-terminal conductor [Eq. (2.32)] along Martin and Landauer [90]. In their paper the fluctuating currents are represented as a result of random transmission of electrons from one terminal to the other. Different processes contribute to the noise for each energy E and mode n:

1. A current pulse occurs whenever an electron wave packet incident from the left terminal is scattered into an empty state in the right terminal. The rate τ_{rl}^{-1} of these events is proportional to the probability $f_L(E)$ for an energy state E in the left reservoir to be occupied times the probability $1 - f_R(E)$ for the state in the right reservoir, in which the electron scatters into, to be unoccupied, times the transmission probability from the left to the right: $T_n^{rl}(E) \equiv T_n(E)$:

$$\tau_{rl}^{-1} \sim f_L(E)[1 - f_R(E)] T_n(E).$$
 (C.1)

The factor $1 - f_R(E)$ ensures that the Pauli principle is fulfilled.

2. Of course the reverse process that electrons scatter from an occupied state in the right reservoir to an unoccupied state in the left reservoir

contributes to the noise, too. The rate of these processes is given by:

$$\tau_{lr}^{-1} \sim f_R(E) [1 - f_L(E)] T_n(E),$$
 (C.2)

where $T_n^{lr}(E) = T_n^{rl}(E) \equiv T_n(E)$ has been taken into account.

Since we require an expression for the fluctuations, i.e. the deviations from the mean current, the mean current squared, which is proportional to $T_n(E) [f_L(E) - f_R(E)]$, has to be subtracted. Thus the contribution from electrons at energy E in one specific mode n to the noise is proportional to:

$$f_L(E)[1 - f_R(E)] T_n(E) + f_R(E)[1 - f_L(E)] T_n(E) - [f_L(E) - f_R(E)]^2 T_n^2(E).$$
(C.3)

The coefficient follows from the fact that if no bias is applied (V = 0) the expression for the thermal noise $4k_BTG$ must be recovered:

$$S = 4k_BTG = 4k_BT \frac{2e^2}{h} \int dE \left(-\frac{\partial f}{\partial E}\right) \sum_n T_n(E)$$
$$= 2\frac{2e^2}{h} \int dE \, 2f(E)[1-f(E)] \sum_n T_n(E). \tag{C.4}$$

In equilibrium $(V = 0) f_L(E) = f_R(E) \equiv f(E)$. Thus the expression in (C.3) equals

$$2f(E)[1 - f(E)] T_n(E).$$
 (C.5)

Comparing Eq. (C.4) and Eq. (C.5) the general expression for the shot noise of a two-terminal conductor (in the zero frequency limit) follows as [34, 35]:

$$S = 2 \frac{2e^2}{h} \sum_n \int dE \left\{ f_L(1 - f_R) T_n + f_R(1 - f_L) T_n - [f_L - f_R]^2 T_n^2 \right\}$$

= $2 \frac{2e^2}{h} \sum_n \int dE \left\{ [f_L(1 - f_R) + f_R(1 - f_L)] T_n(1 - T_n) + [f_L(1 - f_L) + f_R(1 - f_R)]^2 T_n^2 \right\}$ (C.6)

C.2 Vacuum tubes

Figure C.2(a) shows the schematics of a vacuum tube (triode): The heated cathode (K) made of a wounded tungsten wire boils off electrons into a vacuum. These are attracted by the positively charged anode (plate) (Edison

C.2 Vacuum tubes



Figure C.1: Vacuum tubes: (a) Schematics of a triode. Electrons having enegies larger than the work function W of tungsten are emitted from the heated cathode (K), travel through the vacuum and are attracted by the positive anode (A). (b) Photograph of a historical tetrode (triode with additional grid) containing 4 electrodes (Telefunken EL 153).

effect). A grid (or many grids) between cathode and anode, which is negatively charged, controls the electron current. By designing the cathode, grid(s) and plate properly, the tube will make a small AC signal voltage into a larger AC voltage, thus amplifying it [119].

In case that the anode is floating no net current will flow from the cathode to the anode [Fig. C.2(a)]. Instead a negative space-charge is formed in front of the cathode, originating from evaporated electrons which are hold back by the ionized atoms. The size χ of the space-charge region can be calculated solving the Poisson-equation $\Delta \varphi(x) = -en(x)/\epsilon_0$ for the electrical potential $\varphi(x)$ with the electron density $n(x) = n_0 \exp(-e\varphi(x)/k_B\theta) \simeq$ $n_0 [1 - e\varphi(x)/k_B\theta]$, where n_0 is the electron density within the cathode:



Figure C.2: (a) Space-charge region formed in front of the cathode in a opencircuited tube. (b) For sufficiently high bias voltages V the space-charge is removed (saturation regime) and the potential drops linearly. W denotes the work function.

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$$\chi = \sqrt{\frac{\epsilon_0 k_B \theta}{n_0 e^2}}.$$
 (C.7)

The higher the temperature the larger the space-charge region.

When the circuit is closed and the cathode is kept on a higher temperature than the anode, a thermionic current will flow from the cathode to the anode. The magnitude of this current is limited by the negative spacecharge emitted by the cathode. This is also true when the anode is kept at a positive potential with respect to the cathode, because the electrons near the cathode act as a screen and tend to prevent the field due to the anode from being felt in the region near the cathode. In this regime, where the current is limited not by emission but by the space-charge it is given by

$$I = \frac{\sqrt{2}}{9\pi} \sqrt{\frac{e}{m}} \frac{V^{3/2}}{L^2}$$
(C.8)

with L the distance between cathode and anode [119]. Only when the bias voltage V is sufficiently large all electrons are attracted by the anode and the space-charge region is removed. In this case, the current saturates (does no longer depend on the anode voltage) and is determined by the temperature of the cathode [Fig. C.3].

In the space-charge limited regime where the possibility of escape of an electron is limited by the repulsion of the electrons in the space (Coulomb interaction) shot noise is suppressed. Full shot noise $S = S_{Poisson} = 2e|I|$ is only present in the saturation regime [28]. The question whether the shot noise in the saturation regime is classical or quantum in nature is discussed in sect. C.3. Before the electrical field and current in the saturation regime will be estimated.



Figure C.3: Current-voltage characteristics of a vacuum tube illustrating the 3/2-power law [Eq. (C.8)] and the saturation point (S). The dashed curves are IV-curves for different grid voltages. Within the saturation regime the current does no longer depend on the anode voltage because all electrons emitted by the cathode are collected at the anode.

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Figure C.4: Image potential determining the barrier shape in emission of electrons from the hot cathode for different electrical fields \mathcal{E} . For very high fields the barrier becomes very thin so that electrons can tunnel through and the noise is quantum.

C.2.1 Electrical field and current in the saturation regime

At the edge of the vacuum barrier the electron density is approximatively given by $n_0 = a^{-3} \exp(-W/k_B\theta) \simeq 5 \cdot 10^{16} \text{ m}^3$ with $a \simeq 1.2$ Å the typical interatomic distance, W = 4.5 eV the work function of tungsten and the cathode temperature $\theta = 2000$ K. The size χ of the space be-charge region follows from (C.7) and is of the order 10 μ m. The charge build up at the cathode corresponds to an electrostatic surface potential of $k_B\theta/e$ so that the surface electric field can be estimated as $\mathcal{E} \simeq k_B\theta/e\chi$. Inserting numbers the saturation field is of the order 10^4 V/m .

The electrical current density due to thermionic emission from a heated conductor is given by the Richardson-Dushman equation [120]:

$$j = \mathcal{L} \theta^2 \exp(-W/k_B \theta) \tag{C.9}$$

with $\mathcal{L} = emk_B^2/2\pi^2\hbar^3 = 120 \text{ AK}^{-2}\text{cm}^{-2}$. This expression is only correct if the electrical field \mathcal{E} is high enough so that the space-charge is removed (saturation regime). The saturation field \mathcal{E} combined with the potential formed by the holes in the (planar) cathode (image-potential) leads to an electrical potential $\phi(x)$ given by

$$\phi(x) = -\mathcal{E}x - \frac{e}{4\pi\epsilon_0}\frac{1}{x},\tag{C.10}$$

which is illustrated in Fig. C.4. The maximum of $\phi(x)$ lies at $x_0 = \sqrt{e/4\pi\epsilon_0\mathcal{E}}$, where the barrier is lowered by $e\phi(x_0) = -2e\sqrt{e\mathcal{E}/4\pi\epsilon_0} \simeq -8$ meV. This is negligible in comparison with the work function W = 4.5 eV, so that the saturation current can be estimated disregarding the barrier lowering. For a cathode area of 10^{-2} cm⁻² and $\theta = 2000$ K the *emission current* is of the order 10 μ A.

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In the saturation regime, where no space-charge region exists at the cathode and $f_R = f_{anode} = 0$, the shot noise power due to emission of electrons from

the cathode is given according to Eq. (C.6) by

$$S = 2\frac{2e^2}{h}\sum_{n}\int dE \left\{\underbrace{f_{cathode}T_n(1-T_n)}_{quantum} + \underbrace{f_{cathode}T_n^2}_{classical}\right\}.$$
 (C.11)

Here we made use of the fact that the occupation of the hot cathode is small (classical): $f_L = f_{cathode} = \exp(-E/k_B\theta) \ll 1$. Therefore $f_L(1 - f_L) \simeq f_L$. The current I due to emission at the cathode equals

$$I = \frac{2e}{h} \sum_{n} \int dE f_{cathode} T_n \tag{C.12}$$

There are two terms in Eq. (C.11) contributing to the noise: the first term is the quantum mechanical part since it only contributes for transmission probabilities $T \neq 0, 1$. The second term is classically because it dominates when all transmission coefficients are classical, i.e. 0 or 1.

Classical: Because all T_n are 1, $T_n^2 = T_n$ and the shot noise is given by

$$S = 2\frac{2e^2}{h} \sum_{n} \int dE f_{cathode} T_n.$$
(C.13)

The current is

$$I = \frac{2e}{h} \sum_{n} \int dE f_{cathode} T_n \tag{C.14}$$

so that the Fano factor $F \equiv S/2e|I|$ follows as

$$F = \frac{1}{2e} \frac{4e^2}{h} \frac{h}{2e} = 1$$
 (C.15)

which is Schottky's formula S = 2e|I|.

Quantum: If all T_n are small $(T_n \ll 1)$ the noise is due to tunneling (quantum diffraction). In this case the quantum term $\sim T_n$ in Eq. (C.11) dominates while terms proportional to T_n^2 are negligibly small so that the noise is given by

$$S = 2\frac{2e^2}{h} \sum_{n} \int dE f_{cathode} T_n.$$
(C.16)

For the current we have the same expression (C.14) as in the classical case what leads to a Fano factor of 1 as before. We again obtain Schottky's formula S = 2e|I|, this time however, originating from quantum diffraction.

Thus, in order to understand the nature of shot noise in vacuum tubes the transmission probabilities T_n have to be evaluated.





Figure C.5: Transmission probability vs. energy according Eq. (C.17) for different saturation fields \mathcal{E} .

C.3.1 Transmission probability at the cathode

The quantum-mechanical transmission probability is given by [121]

$$T \simeq \left[1 + e^{-2\pi E/\hbar\omega_0}\right]^{-1} \tag{C.17}$$

for electrons with energy E above the barrier [Fig. C.5]. This energy is given by $k_B \theta$ with the cathode temperature θ . ω_0 denotes the negative curvature at the barrier top and determines whether the barrier is sharp or smooth. It can be obtained from the 'force-constant'

$$f = e \left| \frac{\partial^2 \varphi}{\partial x^2} \right|_{x=x_0} = 2\sqrt{4\pi\epsilon_0 e} \,\mathcal{E}^{3/2} \tag{C.18}$$

with $\omega_0 = \sqrt{f/m}$:¹

$$\omega_0 = \left(\frac{16\pi\epsilon_0 e}{m^2}\right)^{1/4} \cdot \mathcal{E}^{3/4}.$$
 (C.19)

If $\hbar\omega_0 \ll k_B\theta$, T = 1 and the classical part of the shot noise in (C.11) dominates. In the other limit $\hbar\omega_0 \gg k_B\theta$ the transmission T is very small and the shot noise is due to tunneling (quantum diffraction).

First rough shot noise measurements in vacuum tubes were carried out by Hartmann in 1921 [27]. A very careful study of the 'Schroteffekt' was performed by Hull and Williams in 1925 [28].

In the first part of this experiment shot noise was measured in the saturation regime, where the thermionic current is limited by temperature². The corresponding parameters are given in the first two lines of Tab. C.1. In this regime the full Schottky-noise 2e|I| (F = 1) has been measured in excellent agreement with Millikan's value for the electron charge e. The

¹This is analogous to the mechanical oscillator $\ddot{x} + \omega_0^2 x = 0$ with F = -fx and $E_{pot} = \frac{1}{2}fx^2$.

 $^{^{2}}$ In order to avoid space-charge a very strong electric field at the cathode surface was achieved by connecting anode plate and grid together [see Fig. C.1].

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\mathcal{E} [V/m]	V_G [V]	V_P [V]	$i_0 \; [mA]$	θ [K]	$\hbar\omega_0/k_B\theta$	Т	F
$3\cdot 10^6$	120	120	1	1675	3.210^{-2}	1	1.00
$3\cdot 10^6$	120	120	5	1940	2.710^{-2}	1	1.00
$1 \cdot 10^4$	-6	130	1	1675	4.410^{-4}	1	0.93
$1\cdot 10^4$	-6	130	3	1805	4.110^{-4}	1	0.49
$1\cdot 10^4$	-6	130	5	1940	3.810^{-4}	1	0.20

Table C.1: Experimental parameters from shot noise measurements of Hull and Williams in 1925 [28]. V_G is the voltage at the grid and V_P at the anode plate. i_0 is the thermionic current. F = S/2e|I| denotes the Fano factor. The second last column shows that the shot noise observed in this experiment is a classical phenomenon.

ratio $\hbar\omega_0/k_B\theta \ll 1$ so that the transmission is 1. Therefore the shot noise observed in this experiment is *classical*.

In the second part of the experiment the effect of the space-charge on the shot noise was investigated at lower electric fields \mathcal{E} . The corresponding parameters are given in the last three lines of Tab. C.1. At lower temperatures the emission current is limited by temperature and the full Schottky-noise is observed. For higher temperatures however the space-charge is increased and the shot noise is gradually suppressed due to Coulomb interaction.

In conclusion it has been shown that the shot noise observed in vacuum tubes is *classical*, what is in profound difference to shot noise observed in mesoscopic conductors.

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Appendix D

Barrier Noise with heating

The following c-code calculates the shot noise for a series N QPCs for the case of inelastic ee-scattering within the cavities in between the contacts. The program allows to vary the transmission of the first two modes between 0 and 1. Thereby, all contacts are treated symmetrically.

```
/* NUMERICAL CALCULATION OF BARRIER NOISE WITH HEATING */
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <math.h>
#define num_kT 50
                           /* limits of integrations */
#define VoltageSteps 5
                           /* voltage steps */
#define intrv 1000
                           /* integration steps */
#define k 1.380658E-23
                            /* k Boltzmann */
#define e 1.602E-19
                            /* electron charge */
#define Co 7 71011
#define Co 7 71011
                            /* resistance quantum */
#define Go 7.74646543918E-05 /* conductance quantum */
int answer,i,j,n,m,N;
float NBarriers;
                            /* number of barriers */
float a,b,c,d;
                            /* dummy variables */
float T0=0.27;
                            /* bath temperature */
float T[50],mu[50];
                            /* temp./chem.pot. (V) */
float E_F=7.5e-3;
                            /* Fermienergy in Volts */
float low=0,up=0;
float e1,e2,e3,e4,e5,st=0;
float IntA=0,I1A,I2A,I3A,I4A,I5A;
float IntB=0,I1B,I2B,I3B,I4B,I5B;
```

```
D Barrier Noise with heating
```

```
float I_max,V_max,I,V;
float Ga1A,Ga2A,Ga1B,Ga2B,RA,GA,RB,GB;
float R,G,Sn,S,fanofactor;
char File_par[16],File_out[16];
FILE *par,*out;
float qua (float x)
   {double res;
   res=exp(2*log(x));
   return res;}
float f (float x, float mu, float T)
   {double res;
   res=1/(1+exp((x-mu)/(k*T)));
   return res;}
main()
{
   E_F=E_F*e;
   printf("\tNumerical Integration to calculate noise\n");
   do
   {
      I_max=0,V_max=0,I=0,V=0;
      Ga1A=0,Ga2A=0,Ga1B=0,Ga2B=0;
      printf("LogFile:\t\t"); scanf("%s",File_par);
      par=fopen(File_par,"w");
      printf("OutputFile:\t\t"); scanf("%s",File_out);
      out=fopen(File_out,"w");
      fprintf(out,"N\tI\tS\tF\n");
      printf("barrier number (2-...)\t"); scanf("%i",&N);
      NBarriers=N;
      printf("transm.ch1 QPC A (0-1)\t"); scanf("%f",&Ga1A);
      printf("transm.ch2 QPC A (0-1)\t"); scanf("%f",&Ga2A);
      printf("transm.ch1 QPC B (0-1)\t"); scanf("%f",&Ga1B);
      printf("transm.ch2 QPC B (0-1)\t"); scanf("%f",&Ga2B);
      RA=Ro/(Ga1A+Ga2A); GA=Ro*(Ga1A+Ga2A);
      RB=Ro/(Ga1B+Ga2B); GB=Go*(Ga1B+Ga2B);
      G=(GA+GB)/2; R=(RA+RB)/2;
      printf("R_tot\t\t%e Ohm\n",N*R);
      printf("I_max (nA)\t\t"); scanf("%f",&I_max);
      I_max=I_max/1e9; V_max=I_max*N/G;
      printf("Vleft-Vright\t\t%e V\n",V_max); printf("\n");
```

```
/* tempearature and chemical potential */
      for (m=0; m<VoltageSteps+1; m++)</pre>
      {
         Sn=0,S=0,fanofactor=0;
         I1A=0,I2A=0,I3A=0,I4A=0,I5A=0;
 I1B=0,I2B=0,I3B=0,I4B=0,I5B=0;
         V=m*V_max/VoltageSteps;
                                       /* actual voltage */
         I=m*I_max/VoltageSteps;
         fprintf(par,"%e\t\n",I);
         printf("temp. and chem.pot.for I=%f nA\n",I/1e-9);
         printf("n\tN\t\tT[n] (K)\tmu[n] (eV)\n");
         for (n=0; n<N+1; n++)</pre>
         {
            b=n;c=N;a=b*(c-b)/(c*c);
            printf("%i\t%i\t",n,N);
            T[n]=sqrt(T0*T0+4.0962680e7*a*V*V);
            mu[n]=V*(1-b/c);
            printf("\t%f\t%e\n",T[n],mu[n]);
            fprintf(par,"%e\t%e\n",T[n],mu[n]);
            mu[n]=mu[n]*e;
                                 /* voltage to energy */
         }
         fprintf(par,"\n");
/* integration */
         printf("* wait for noise calculation *\n");
         for (n=0; n<N; n++)</pre>
                               /* adjust limits */
         ł
low=-num_kT*k*(T[n]+T[n+1])/2+(E_F+(mu[n]+mu[n+1])/2);
up = num_kT*k*(T[n]+T[n+1])/2+(E_F+(mu[n]+mu[n+1])/2);
            st=(up-low)/intrv;
            IntA=0; e1=0,e2=0,e3=0,e4=0,e5=0;
            for (i=0; i<intrv+1; i++)</pre>
            {
               e1=st*i+low;
       e2=st*(4*i+1)/4+low;
               e3=st*(2*i+1)/2+low;
       e4=st*(4*i+3)/4+low;
               e5=st*(i+1)+low;
I1A=qua(f(e1,E_F+mu[n],T[n])-f(e1,E_F+mu[n+1],T[n+1]))*st;
I2A=qua(f(e2,E_F+mu[n],T[n])-f(e2,E_F+mu[n+1],T[n+1]))*st;
I3A=qua(f(e3,E_F+mu[n],T[n])-f(e3,E_F+mu[n+1],T[n+1]))*st;
I4A=qua(f(e4,E_F+mu[n],T[n])-f(e4,E_F+mu[n+1],T[n+1]))*st;
```

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```

```
D Barrier Noise with heating
```

```
I5A=qua(f(e5,E_F+mu[n],T[n])-f(e5,E_F+mu[n+1],T[n+1]))*st;
IntA=IntA+(I1A+I2A+I3A+I4A+I5A)/5;
IntB=IntB+(I1B+I2B+I3B+I4B+I5B)/5;
           }
                               /* END Integration loop */
Sn=Sn+Go*(k*(T[n]+T[n+1])+(Ga1A*(1-Ga1A)+Ga2A*(1-Ga2A))*IntA);
        }
                               /* END n-loop */
        S=2*Sn/qua(NBarriers); /* noise */
        printf("current noise\t\t%e\n",S);
        fanofactor=S/(2*e*I); /* fanofactor */
        printf("fanofactor\t\t%e\n",fanofactor); printf("\n");
      }
                                   /* END m-loop */
     fprintf(out,"%i\t%e\t%e\n",N,I,S,fanofactor);
     fclose(out);
     fclose(par);
     printf("continue (no='0')\t"); scanf("%i",&answer);
  }
while (answer>0);
                           /* END main() */
}
```

```
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```

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- The Fermionic Hanbury Brown and Twiss Experiment, Meeting of the Swiss Physical Society, 13. February 1999, Bern, Switzerland.
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- Shot Noise by Quantum Scattering in Chaotic Cavities, 36th Rencontres de Moriond, "Electronic Correlations: From meso- to nanophysics", 20.-27. January 2001, Les Arcs, France.
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Figure 10.1: From a fruitful discussion with Christian about various universal Fano factors like $1/2, 1/3, 1/4, \ldots$

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